UNIT-II

Sets : Definition of sets, representation of sets, type of sets, Operations on sets, Sub sets, Power set, Universal set, Complement of a set, Union and Intersection of two sets, Venn diagrams, De-Morgans law of sets, Partition of sets, Duality Principles.

Relations: Relation, Types of relations- reflexive, symmetric, anti-symmetric, transitive equivalence and partial order relation. Relation and diagraphs, Cartesian product of two sets. Functions: Function, domain and range, One to one and onto functions, composite functions, inverse of a functions. Binary operations

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Sets, Relation & Functions

Set:-Collection of well defined elements is known as set. Well defined means for any object, we can identify with confirmation that object is element of collection or not.

Here, collection may be any type of objects. Generally Set is represented by Capital letters of english alphabate.

Eg:-

A={1,2,3,4,5}

B={x,y,z}

C={Ram,Mohan,Ravi}

D=Rivers of india

E=Students of Agarwal classes

Order of a Set:-

Number of elements in a Set is known as order of a set. For any Set A, it is represented by O(A)

(i.e. order of a Set A)

eg. A={1,2,x,y}

O(A)=4

B={1,2,3,4,5}

O(A)=5

C={4}

O(C)=1

D={}

O(D)=0

E={0}

O(E)=1

F={ ϕ }

O(F)=1

G= {1, 2, 3, ...n}

O(G) = ∞

H=Students of Agarwal classes

O(H)= Total no of Students in Agarwal classes.

Type of Set based on order of a Set :-

1. Singleton Set:- A Set which contain only one element.

i.e. O(A)=1

e.g. A={x}

2. Null Set/void Set/Empty Set :- A Set which contain no element or Zero element is knowm as null set.

It is represented by {} or ϕ

null O(A)=0

A={}=ϕ

3. Finite Set:- If a set contain finite number of elements

i.e. O(A)= finite number

eg. A={1,2}

B={1}

C={}

D= River of india

E=Students of Agarwal classes

4.Infinite Set:-

If a Set contain infinite number of element.

i.e. O(a)= ∞

eg.. A={2,4,6,8,10............n}

B=Stars in the sky

5.Equivalent set:- Two sets A&B are known as equivalent set if

O(A)=O(B)

i.e. Both Set contains equal numbers of elements

e.g. A={1,2,3}

B={x,y,z}

O(A)=3, O(B)=3

$\Rightarrow$ O(A)=O(B)

So A & B are equivalent Sets

It is represented by

A $\equiv$ B

i.e. Set A is equivalent to B

eg. A={1,2,5},B={3,4,6,7}

O(A)=3, 0(B)=4

O(A) $\neq$ 0(B)

So A&B are not equivalance set

It is represented by A $\not\equiv$ B

(Ais not equivalance to B)

6. Equal Set :- Two set A& B are known as equal se if number of element in both sets and also elements be same in both sets

eg.:-

A={1,2,5}

$B=\{1,2,5\}$

here $O(A)=O(B)$

& also element are same so A&b are equal sets.

It is represented by

$A=B$

i.e. set A is equal to B

eg. $A=\{1,2,3\}$

$B=\{2.3.1\}$

$A=b$

eg. $A=\{x,y,z,p\}$

$B=\{x,y,z\}$

$A\neq B$

eg. $A=\{x,y,z\}$

$B=\{a,b,c\}$

$A\neq B$

Note:-

Every equal set are also equivalentsets but not vice-versa. (i.e. Every equivalent sets are not equal sets.)

Symbols used in set Theory :-

$\in \rightarrow$ belong to or exist in or member of

eg. $a \in A \Rightarrow$ element a belong to set A or elements a is exist in set A.

$\notin \rightarrow$ does not to or not exist in or is not member of

eg. $b \notin A$

$\Rightarrow$ element 'b' is not exist in set A.

$\forall \rightarrow$ for all

$\exists \rightarrow$ there exist

$N \rightarrow$ Set of Natural no.

$\{1,2,3,4,\ldots\ldots\}$

$w \rightarrow$ Set of whole No.

$\{0,1,2,3,4,\ldots\ldots\}$

$I \rightarrow$ Set of Integers

$\{\ldots\ldots-3,-2,-1,0,1,2,3\ldots\ldots\}$

$R \rightarrow$ Set of Real Nos

$Q \rightarrow$ Set of Rational nos.

$U \rightarrow$ Universal Set

(When used as Set)

$\cup \rightarrow$ Union

(When used as operator)

$\cap \rightarrow$ Intersection

$- \longrightarrow$ Set Difference

$\oplus \rightarrow$ Symmetric Difference

$\bar{A} \rightarrow$ Complement of set A

$\times \rightarrow$ Cartation product of two sets.

Set Representation Methods :-

We can represent a set in two manners:-

1. Roster form or tabuler form
2. Rule form or set builder form

1.Roster forms-

Write all elements of set in curly braces{} by seperating then using commas.

eg. $A=\{1,2,3,4\}$

$B=\{2,4,6,8,10\}$

Rule form :-

Here, we write a genral rule by which all elements of set can be genrated.

eg. :- $A=\{1,2,3,4\} \rightarrow$ Roster form

$\Downarrow$ Rule Form

$A=\{x|x \leq 4; x \in N\}$ Rule form

$B=\{0,1,2,3\}$

$B=\{x|x \leq 3, x \in w\}$

$C=\{2,4,6,8,10\}$

$C=\{x|x=2n \& n \leq 5 ; n \in N\}$

$D=\{1,3,5,7,9,11,13\}$

$D=\{x|x=2n-1 . \& n \leq 7, n \in N\}$

$E=\{7,14,21,28\}$

$E=\{x|x=7n \& n \leq 4, n \in N\}$

$F=\{5,6,7,8,9\}$

$F=\{x|5 \leq x \leq 9, x \in N\}$

$G=\{2,4,6,8,10..............\}$

$G=\{x|x=2n, n \in N\}$

$H=\{1,3,5,7,9,11............\}$

$H=\{x|x=2n-1, n \in N\}$

Sub set :-

Any set A is known as sub set of any set B if every element of set A is also exiest in set B i.i If set B contains all elements of Set A then A is known as subset of B

It is represented by $A \subset B$

CA is Subset of B

Eg

$A=\{2,4,5\}$

$B=\{2,4,5,6,7\}$

$A \subseteq B$

Eg

$A=\{1,3,7,9\}$

$B=\{1,2,3,7,8,10,11\}$

$A \subseteq B$

Q. Find all possible subset of A = {a,b,c}

$B=\{a\}$

$C=\{b\}$

$D=\{c\}$

$E=\{a,b\}$

$F=\{a,c\}$

$G=\{b,c\}$

$H=\{a,b,c\}$

$I=\{\}$

Subset

Proper Subset | Inproper Subset

Improper Subset :- For any set A, null set and the set itself is always subset of A. So these two subset$\{\phi, A\}$ are know as inproper subset.

Proper Subset :-

Remainaing (i.e. excluding ϕ & A) all possible subset of A are proper subset.

Eg $A=\{0,1\}$

$\underbrace{\{0\},\{1\},}_{\uparrow} \underbrace{\{0,1\}, \{\}}_{\uparrow}$

Proper subset, Inproper subset

Eg $B=\{0,1,2\}$

$\left.\begin{array}{l}\{0\},\{1\},\{2\},\{0,1\},\\ \{1,2\},\{0,2\}\end{array}\right]$ Proper subset

$\{0,1,2\},\{\}$ → Inproper subset

Power Set :-

Power set of a set A is a collection of all possible subset of set A

It is represented by P(A)

i.e power set of set A

(It contains all possible subset of A)

Eg :-

$A=\{0,1\}$

$p(A)=\{\{0\},\{1\},\{0,1\},\{\}\}$ $O(p(A))=4$

Eg

$B=\{0,1,2\}$

$p(B)=\{\{0\},\{1\},\{2\},$
$\{0,1\},\{1,2\},\{0,2\},$
$\{0,1,2\},\{\}$ $O(p(A))=8$

e.g. $C=\{a,b,c,d\}$

$P(C)=\{\{\},\{a\},\{b\},\{c\},\{d\},\{a,b\},$
$\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\},\{a,b,c,d\}\}$

Note:- if $O(A)=n$

then $O(P(A))=2^n$

Operation on Set :- Following operators may be applied on two sets A&B

1. Union AUB
2. Intersection $A\cap B$
3. Set difference A-B &B-A

4. Symetric difference $A\oplus B$

1.Union :-

The union of two sets A&B is a set AUB that contains elements of A or and B (i.e. all elements of A&B) i.e.

i.e. $AUB=\{x|x\in A \text{ or . and } x\in B\}$

Eg

$A=\{1,2,9,11,12\}$

$B=\{5,9,12,17,x,y\}$

$AUB=\{1,2,9,11,12,5,17,x,y\}$

2. Intersection :-

The intersection of two sets A&B contain common elements of A&B (i.e. the elements which are in both A&B

$A\cap B=\{x|x\in A \text{ and } x\in B\}$

Eg. :-

$A=\{1,2,3,4,7,10\}$

$B=\{x,y,4,2,7,9,10,11\}$

$A\cap B=\{2,4,7,10\}$

3.Set difference :-

The set difference of two sets A&B is a Set A-B that contains element of A but not of B(i.e. A-B contain elements of only A)

$A-B=\{x|x\in A \text{ and } X\notin A \text{ and } x\notin B^2\}$

$B-A=\{x|x\in B \text{ and } x\notin A\}$

B-A set contains elements of only B (i.e. elements of B but not of A)

$A-B\neq B-A$

eg.:-

$A=\{2,4,6,x,y\}$

$B=\{1,4,5,6,x,z\}$

$A-B=\{2,y\}$

$B-A=\{1,5,z\}$

$A-B=A-(A\cap B)$

$B-A=B-(A\cap B)$

4. Symmetric difference :-

The symmetric difference of two sets A&B is a set $A\oplus B$ that contains elements of A or elements of B but not common elements of A&B (i.e. $A\oplus B$ contain elements of only A or only B.

$A\oplus B=\{x|x\in A \text{ and } x\notin B)$

or

$(x\in B \text{ and } x\notin A)\}$

(i.e. either A or B but not Both)

$A\oplus B=(A-B)\cup(B-A)$

$A\oplus B=(A\cup B)-(A\cap B)$

eg.

$A=\{2,4,6,x,y\}$

$B=\{1,4,5,6,x,z\}$

$A\oplus B=\{2,y,1,5,z\}$

eg. $A=\{2,4,5,7,9\}$

$B=\{x,y,p,7,9,5,3\}$

$A\cup B=\{2,4,5,7,9,x,y,p,3\}$

$A\cap B=\{5,7,9\}$

$A-B=\{2,4\}$

$B-A=\{x,y,p,3\}$

$A\oplus B=\{2,4,x,y,p,3\}$

eg. $A=\{2,5,7,9\}$

$B=\{2,9,7,5\}$

$A\cup B=\{2,5,7,9\}$

$A\cap B=\{2,5,7,9\}$

$A-B=\{\}$

$B-A=\{\}$

$A\oplus B=\{\}$

eg. $A=\{2,5,6\}$

$B=\{\}$

$A\cup B=\{2,5,6\}$

$A\cap B=\{\}$

$A-B=\{2,5,6\}$

$B-A=\{\}$

$A\oplus B=\{2,5,6\}$

Eg.

$A=\{\}$

B= {5,7,x,क}

A∪B={5,7,x,क}

$A\cap B=\{\}$

$A-B=\{\}$

B-A = {5,7,x,क}

A⊕B= {5,7,x,,क}

Eg.

$A=\{2,6,9\}$

$B=\{1,5,x,p\}$

$A \cup B = \{2,6,9,1,5,x,p\}$

$A \cap B = \{\}$

$A - B = \{2,6,9\}$

$B - A = \{1,5,x,p\}$

$A \oplus B = \{2,6,9,1,5,x,p\}$

Eg

$A = \{x \mid x = 2n, n \in N\}$

$B = \{x \mid x = 2n-1, n \in N\}$

$A \cup B = \{x \mid x = n, n \in N\}$

$A \cap B = \phi$

$A - B = \{x \mid x = 2n, n \in N\}$

$B - A = \{x \mid x = 2n-1; x \in N\}$

$A \oplus B = N$

Eg

$A = \{x \mid x \leq 9, x \in w\}$

$B = \{x \mid 4 \leq 8, x \in N\}$

$A \cup B = \{x \mid x \leq 9, x \in w\}$

$A \cap B = \{x \mid 4 \leq x \leq 8, x \in N\}$

$A - B = \{x \mid x \leq 3 \text{ or } x = 9, x \in w\}$

$B - A = \{\}$

$A \oplus B = \{x \mid x \leq 3 \text{ or } x = 9, x \in w\}$

Unordered Set :-

If there is no importance of order of elements in a sets.

Eg

$A = \{2,4,8,10\}$

$B = \{4,8,2,10\}$

$A = B$

Ordered set:-

When set elements are in pair form and there order is important. If we change the order then element be also changed.

eg

Co-Ordinate of a point (x,y)

$A = \{(5,7),(10,15),(30,15)\}$

Cartesion product of two sets :-

The cartesion product of two set A&B is an ordered set A*B that contain elements in pair form. In each pair first elements is of set A & Second element is of set B.

$A * B = \{(x,y) \mid x \in A, y \in B, \forall x \in A, \forall y \in B\}$

eg

$A = \{1,2,3\}$

$B = \{X,y\}$

$A*B = \{(1,x),(1,y),(2,x),(2,y),(3,x),(3,y)\}$

$B*A = \{(x,1),(x,2),(x,3),(y,1),(y,2),(y,3)\}$

$A*B \neq B*A$

$O(A*B) = O(A) \times O(B)$

$= 3 \times 2 = 6$

If A or B is null set then A*B or B*A is also null Set.

Note:-

$O(A*B) = O(B*A)$

eg.

$A = \{1,2,3,4,5,\ldots\ldots\ldots\ldots\}$

$B = \{a,b\}$

$A*B = \{(1,a),(2,a),(3,a)$.........

$(1,b),(2,b),(3,b),$..........$\}$

ie. if only one set eithe A or B is infinite set then A*B & B*A is also infinite set.

Universal set (U)

Any Set U is known as universal set of some given set (like A,B&C)

if it contain all elements of the given set like (A,B&C) and there may be some extra elements other than of the given set (like A,B &C)

ie. If the given sets (like A,B& C) ae subset of set U then set U is known as Universal Set for the given Set (like A,B,&C)

Eg.

Final a universal Set for Following Set :-

$A=\{2,5,9,10\}$

$B=\{1,x,3,5,11\}$

$U=\{1,2,3,5,9,10,11,X\}$

or

$U=\{1,2,3,4,5,6,7,8,9,10,11,12,13,p,x,y,z\}$

$A \subseteq U$ $\quad$ $B \subseteq U$

Note :-

Universal Set of some given Set is not unique.

Complement of a Set :-

Complement of a set A is represent by $\bar{A}$ that contains elements which are not of A but exist in the universal Set U.

i.e. $\quad \bar{A} = U-A$

Eg

$A=\{2,3,4,5\}$

$B=\{1,5,9\}$

$U=\{1,2,3,4,5,6,7,8,9\}$

$\bar{A}=\{1,6,7,8,9\}$

$\bar{B}=\{2,3,4,6,7,8\}$

Eg.

$A=\{x|x=2n, n\in N\}$

$B=\{x|x=2n, n\in N\}$

$U=\{x|x=n; n\in N\}$

$\bar{A}=B=\{x|x=2n-1, n\in N\}$

$\bar{B}=A=\{x|x=2n, n\in N\}$

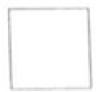
$\overline{A\cap B}=\{x|x=n, n\in N\}$

$\overline{AUB}=\{\}$

Venn Diagram or Area Diagram :-

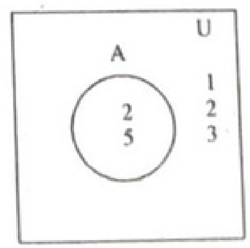
We can represent the set in pictorial or graphical form by using venn diagram.

Symbols :-

Universal Set $\rightarrow$  $\rightarrow$ rectangle

Normal set $\rightarrow$  $\rightarrow$ Circle

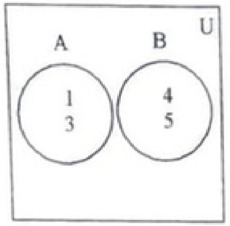
Eg



$U=\{1,2,3,4,5\}$

$A=\{2,5\}$

Eg.



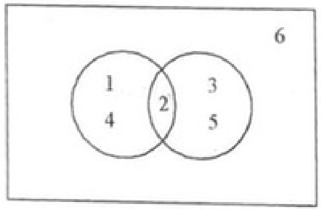
U={1,2,3,4}

A={1,3}

B={4,5}

Here A&B are Disjoint sets (no element is common

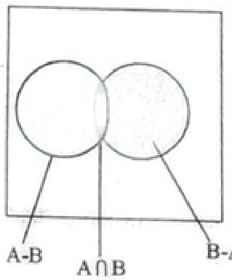
eg.

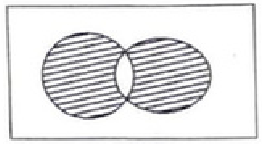


U={1,2,3,4,5,6}

A={1,2,4}

B={2,3,5}





A⊕B

Principal of Inclusion & Exclusion :-

n(AUB) =n(A)+n (B) -n (AB)

n(AUBUC) = n(A)+n(B)+ n(C)-n(AB-n(BC)-n(CA)+n(ABC)

e.g. In a group of 70 students 50 like maths & 40 like DBMS. Find How many students like both maths DBMS.

n(AUB)= n(A)+n(B) -n (AB)

70= 50 + 40-n(AB)

n (AB)= 90-70=20

e.g. In a class of 100 person 70 like Red color, 50like pink color. Find How many persons like only pink color

n(AUB)=n(A)+n(B)-n(AB)

100=70+50-n(AB)

n(AB)=120-100=20

n(B-A)=n(B)-n(AB)

=50-20

=30

n(A-B) = n(A)-n(AB)

=70-20

=50

eg. In a group of 80 people 20 like both Hindi & english and 50 like Hndi find how many Like English.

n(AUB) = n(A+n(B)-n(AB)

$80 = 50 + n(B) - 20$

$\Rightarrow$ $n(B)=50$

$n(B-A)= n(B) - n(A \cap B)$

$= 50-20$

$= 30$

eg. Find how many number are divisible by 3 or 7 between 1 to 100

$n(AUB)=n(A)+n(B)-n(AB)$

$n(AUB)= 33+14-4$

$= 43$

eg. Find how many numbers are divisible by 3 or 5 or 8 betweeb 1to 100

$n(AUBUC) = n(A)+ n(B)+ n(C) - n(AB) - n(BC) - n(AC) + n(ABC)$

$=33+20+12-2-4+0$

$=53$

eg. In a survey of 200persons, it was found. That 70 persons like H.T. 80 persons like TOI and 90 persons like DNA 30 persons like both HT & TOI , 20 persons like both TOI & DNA , 10 persons like all 3 new paper find how many persons like both DNA & HT.

$n(AUBUC) = n(A)+ n(B) + n(C) - n(AB) - n(BC) - n(AC) + n(ABC)$

$\Rightarrow$ $200= 70+80+90-30-20-n(AC) + 10$

$\Rightarrow$ $n(AC) =0$

Multi Set

A Set which allow multiple occurance of element.

eg. A={x, x, 1,2,3,3,4,5,3}

The number of times an element occur is known as multiplicity factor of the element.eg. here multiplicity of x is 2, multiplicity of 3 is 3 Multiplicity of 2 is 1.

eg.

Set of digit in a mobile number like Rahul Agarwal Mobile number is 9460366303 It is a multi set {9,4,6,0,3,6,6,3,0,3}

Eg. **Set of friends :-**

Set of friends of

Ravi = {Mohan, Sohan, Ram, Sohan, Ravi, Ram}

Operation on Multiset :-

Eg

A= {1,2,2,3,4,4,4,6}

B= {1,2,3,3,3,4,4,8,9}

AUB = {1,2,2,3,3,3,4,4,4,6,8,9}

$A \cap B$ = { 1,2,3,4,4}

A-B = {2,4,6}

B-A = (3,3,8,9}

$A \oplus B$ = {2,3,3,4,6,8,9}

Eg

A= {x,x,x,y,p,q,q}

B= { x,y,y,y,p,p,q,q}

AUB= {x,x,x,y,y,y,p,p,p,q,q}

AUB = { x,y,p,q,q}

A-B = {x,x}

B-A= {y,y,p,p}

$A \oplus B$= {x,x,y,y,p,p}

Set Theorems :-

Theorim-1: - State and prove commutative law for union of two Sets :-

AUB =BUA

let $x \in AUB$

$\Rightarrow$ $x \in A$ or $z \in B$

$\Rightarrow$ $z \in B$ or $x \in A$

$\Rightarrow$ $x \in BUB$

So $AUB \subseteq BUA$

Again :-

let $x \in BUA$

$\Rightarrow$ $z \in B$ or $x \in A$

$\Rightarrow$ $x \in A$ or $x \in B$

$\Rightarrow$ $x \in A \cup B$

$\Rightarrow$ $BUA \subseteq AUB$

So $AUB = BUA$

Theorem-2 State and prove De-Morgan's law

$(AUB)' = A' \cap B'$

$(A \cap B)' = A' \cap B'$

Let $x \in (AUB)'$

$\Rightarrow$ $x \notin (AUB)$

$\Rightarrow$ $X \notin A$ and $x \notin B$

$\Rightarrow$ $x \in A'$ and $x \in B'$

$\Rightarrow$ $x \in A' \cap B'$

$(AUB)' \subseteq A' \cap B')$

Let $x \in (A' \cap B')$

$\Rightarrow$ $x \in A'$ and $x \in B'$

$\Rightarrow$ $x \notin A$ and $x \notin B$

$x \notin A \cup B$

$x \in (AUB)'$

So $A' \cap B' \subseteq (AuB)'$

So $(AUB)' = A' \cap B'$

Theorem 3.:- Prove distributive law

$AU(BUC) = (AUB) \wedge (AUC)$

Let $x \in A \cup (B \cup C)$

$\Rightarrow \chi \in A$ or $\chi \in B \cap C$

$\Rightarrow \chi \in A$ or $(\chi \in B$ and $\chi C)$

$\Rightarrow \chi \in A$ or $\chi \in B$ and $\chi \in A$ or $\chi \in C$

$\Rightarrow \chi \in A \cup B$ and $\chi \in A \cup C$

$\Rightarrow \chi \in (A \cup B) \wedge (A \cup C)$

So $A \cup (B \cap C) \subseteq (A \cup B) \wedge (A \cup C)$

Let $\Rightarrow \chi \in (A \cup B)$ and $\chi \in A \cup C$

$\Rightarrow (\chi \in A$ or $\chi \in B)$ and $(\chi \in A$ or $\chi \in C)$

$\Rightarrow \chi \in A$ or $(\chi \in B$ and $\chi \in C)$

$\Rightarrow \chi \in A$ or $(\chi \in B \cap C) \Rightarrow \chi \in A \cup (B \cap C)$

$\Rightarrow (A \cup B) \wedge (A \cup C) \subseteq A \cup (B \cap C)$

So $A \cup (B \cup C) = (A \cup B) \wedge (A \cup C)$

Theorem 4.:

Prove that

$A - (B \cup C) = (A - B) \cap (A - C)$

Proof:- Let $\chi \in A - (B \cup C)$

$\Rightarrow \chi \in A$ but $\chi \notin B \cup C$

$\Rightarrow \chi \in A$ but $(\chi \notin B$ and $\chi \notin C)$

$\Rightarrow (\chi \in A \text{ but } \chi \notin B) \text{ and } (\chi \notin A \text{ but } \chi \notin C)$

$\Rightarrow \chi \in A - B \text{ and } \chi \in A - C$

$\Rightarrow \chi \in (A-B) \wedge (A-C)$

So $A-(B \cup C) \subseteq (A-B) \wedge (A-C)$

Now:-

Let $\chi \in (A-B) \wedge (A-C)$

$\chi \in (A-B) \text{ and } -\chi \in A-C$

$\Rightarrow (\chi \in A \text{ but } \chi \notin B) \text{ and } (\chi \in A \text{ but } \chi \notin C)$

$\Rightarrow \chi \in A \text{ but } (\chi \notin B \text{ and } \chi \in C)$

$\Rightarrow \chi \in A \text{ but } (\chi \notin B \cup C)$

$\Rightarrow \chi \in A-(B \cup C)$

So $(A-B) \wedge (A-C) \subseteq A-(B \cup C)$

So $A-(B \cup C) = (A-B) \wedge (A-C)$

Theoreme 5

Prove that

$A \cup B = A \cap B \Leftrightarrow A = B$

Prove :-

Let $AUB = A \cap B$

let $x \in A$

$x \in AUB$

$x \in A \cap B (\therefore AUB = A \cap B)$

$x \in A \text{ and } x \in B$

$x \in B$

So $A \subseteq B$

Now, Let $x \in B$

$\Rightarrow x \in AUB$

$\Rightarrow x \in A \cap B (\therefore AUB = A \cap B)$

$\Rightarrow x \in A \text{ and } x \in B$

$\Rightarrow x \in A$

So $B \subseteq A$

$A = B$

Reverse :-

Let $A = B$

let $x \in AUB$

$\Rightarrow x \in A \text{ or } x \in B$

$\Rightarrow x \in A \text{ or } x \in A (\therefore A = B)$

$\Rightarrow x \in A \cap A$

$\Rightarrow x \in A \text{ and } x \in A$

$\Rightarrow x \in A \text{ and } x \in B$

$\Rightarrow x \in A \cap B$

So $AUB \subseteq A \cap B$

Now:-

Let $x \in A \cap B$

$\Rightarrow x \in A \text{ and } x \in B$

$\Rightarrow x \in A \text{ and } x \in A$

$\Rightarrow x \in AUA$

$\Rightarrow x \in A \text{ or } x \in A$

$\Rightarrow x \in A \text{ or } x \in B \ (\therefore A = B)$

$\Rightarrow x \in AUB$

$\Rightarrow$ so $A \cap B \subset A \cup B$

$\Rightarrow$ so $A \cup B = A \cap B$

Theorem. 6:

Prove that

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let $(x,y) \in A \times (B \cap C)$

$x \in A$ and $y \in (B \cap C)$

$x \in A$ and ($y \in B$ and $y \in C$)

($x \in A$ and $y \in B$) and ($x \in A$ and $y \in C$)

$(x,y) \in A \times B$ and $(x,y) \in A \times C$

$(x,y) \in (A \times B) \cap (A \times C)$

So $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$

Now $(x,y) \in (A \times B) \cap (A \times C)$

$\Rightarrow$ $(x,y) \in (A \times B)$ and $(x,y) \in A \times C$

($x \in A$ and $y \in B$) and ($x \in A$ and $y \in C$)

$x \in A$ and ($y \in B$ and $y \in C$)

$x \in A$ and $y \in (B \cap C)$

So $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$

So $A \times (B \cap C) = (A \times B) \cap (A \times C)$

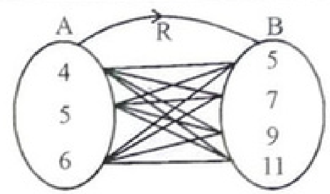
Relation : Relation R associate some elements of set A to some elements of set B.

Eg :-

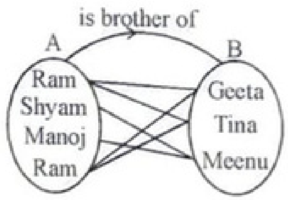
$A = \{4, 5, 6\}$

$B = \{6, 5, 7, 9, 11\}$

$R : A \to B$ suth that $a \le b$, $a \in A$, $a \in B$



Eg :-

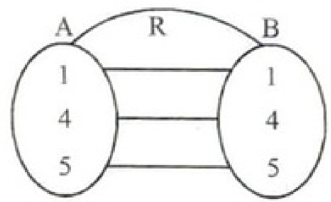


(Pictorial Repusentation)

eg. $A = \{1, 4, 5\}$

$R : A \to A$ s.t. $a = b$, $a \in A$, $b \in A$

Pictorial Representation



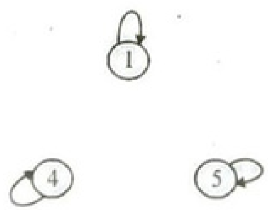
Set Representation :

$R = R(1, 1), (4, 4), (5, 5)$

$(1, 1) \in R$ or $1 R 1$

$(2,1) \notin R$ or $2 \not R 1$

Graph Representation :



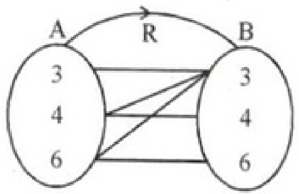
Matrix Representation :

$$\begin{array}{c|ccc} & 1 & 4 & 5 \\ \hline 1 & 1 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 5 & 0 & 0 & 1 \end{array}$$

eg. $A = \{3, 4, 6\}$

$R : A \to A$ s.t. $a > b, a \in A, b \in B$

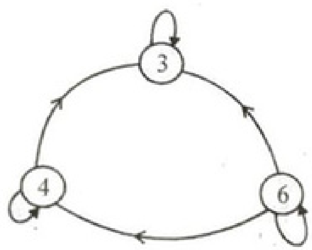
Pictorial form:-



Set form:-

$R = \{(3, 3), (4, 3), (4, 4), (6, 3), (6, 4), (6, 6)\}$

Di Graph form:-



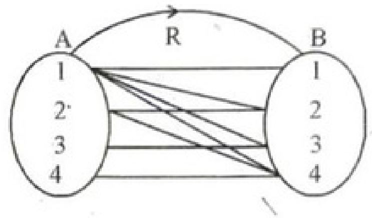
Madrix form:-

$$\begin{array}{c|ccc} & 3 & 4 & 6 \\ \hline 3 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 \\ 6 & 1 & 1 & 1 \end{array}$$

eg. $A = \{1, 2, 3, 4\}$

$R : A \to A$ s.t. a|b; $a \in A, b \in A$

Sol: Pictocial. form:-



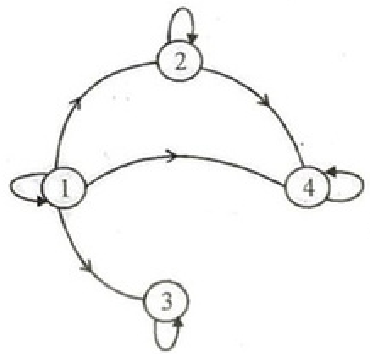
set form:-

$R : f(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)$

Matrix form:-

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 & 1 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

Digraph form:-



Types of Relation :

(1) Reflexive Relation :

$\forall a \in A$

$(a, a) \in R$

(2) Symmetric Relation :

$\forall a, b \in A$

if $(a, b) \in R$

then $(b, a) \in R$

(3) Anti-symmetric Relation :

$\forall a, b \in A$

if $(a, b) \in R$

then $(b, a) \in R$

Eg :-

Set of Parallel lines → Reflexive, symmetric & Isansitive

Set of perpendicular lines → Only symmetric

Eg :

(4) Transitive Relation:-

if (a = b)

$\forall a, b, c \in A$

if $(a, b) \in R$ & $(b, c) \notin R$

Then $(a, c) \in R$

eg. $A = \{a, b, c\}$

Relation	Ref	Sym	Antisymm	Transitive
$R_1 = \{a,b\}, \{b,c\}, (a,a), (b,b), (b, c), (c, c)$	√	×	√	×
$R_2 = \{(a, a), (a, b), (b,b), (b, a)\}$	×	√	×	√
$R_3 = \{(a,a), (a,b), (a,b), (b,b), (c,b), (b,a), (c,c)\}$	√	√	×	×
$R_4 = \{(a,b), (b,a), (b,c)\}$	×	×	×	×
$R_5 = \{(a,a), (b,b), (c,c)\}$	√	√	√	√
$R_6 = \{(b,b)\}$	×	√	√	√
$R_7 = \{(a,b), (b,c)\}$	×	×	√	×
$R_8 = \{\}$	×	√	√	√

eg. $A = \{a, b, c, d\}$

$R = \{(a,a), (a,b), (b,c), (b,b), (d,d), (c,d)\}$

(1) Not Reflexive $\therefore (c,c) \notin R$

(2) Not symaretric $\because$ $(a, b) \in R$ but $(b,a) \notin R$

(3) Not Antisymometric $\because (b, c) \in R$ and $(c, b) \in R$

(4) Not transitive $\because (a,b) \in R$, $(b, c) \in R$, $(a, c) \notin R$

eg. $A = \{4, 6, 7, 9\}$

$R = \{(4,4), (6,6), (7,7), (9,9), (4,6), (6,4), (4,7), (7,9)\}$

Sol. : Reflexive $\because \forall a \in A$ $(a,a) \in R$

Not symmetric – $(4, 7) \in R$ but $(7, 4) \notin R$

Not Autisymmetric – $(4, 6) \in R$ and $(6, 4) \in R$

Not Transitive – $(6, 4) \in R$ and $(4, 7) \in R$ but $(6, 7) \notin R$

eg. $A = \{1, 2, 3, 4, 5, 6\}$

$R : A \to A$ s.t. a|b

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}

R is Reflexive

R is not symmetic

R is Autisymmetric

R is Transitive.

It is known as partically order relation.

Partially order Relation : (P.O.R.)

A Relation R is known as P.O.R. if R is reflexive, and antisymmetric and transitive.

Equivalent Relation : (E.R.)

A relation R is known as E.R. if R is Reflexive, Symmetric and transitive.

eg. $A = \{1, 2, 3\}$

$R:-A \to A$ s.t. $a \le b$, $(a, b \in A)$

Sol. : R = {1, 1}, (1, 3), (1, 2), (3, 3), (2, 3), (2, 2)

R is Reflexive, Antisymmetric and Transitive. So it is P.O.R.

Hasse Diagram :

Hasse diagram is used forgraphical represintation of the P.O.R. relation. This diagram follow Bottom-up approach.

P.O.R. Means :

Reflexive

Antisymmetric

Transitive

It is a refineanent of diagraph representation of the P.O.R. :

Remove loops from diagraph (∵ Reflexive)

Remove transitive edges

Remove direction (∵ bottom to top)

$a\,R\,b \qquad b$

$a \to b \qquad a$

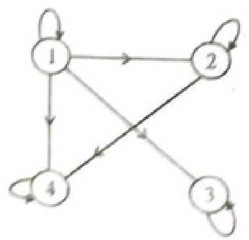
eg. $A = \{1, 2, 3, 4\}$

$R: A \to A$ s.t. a|b; $a, b \in A$

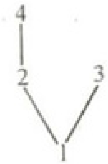
Draw Hasse diagram for this

Sol. : R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}

Diagraph :



Hasse Diagram :

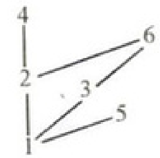


eg. $A = \{1, 2, 3, 4, 5, 6\}$

$R: A \to A$ s.t. a|b, $a, b \in A$

Sol. : R : {(1, 1,), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}

Hasse Diagram :



eg. $A = \{1, 2, 3\}$

$R: A \to A$ s.t. $a = b$, $a, b \in A$

$R = \{(1, 1), (2, 2), (3, 3)\}$

R is P.O.R. so Hosse diagram is.

Hasse Diagram :

1 2 3

Eg. $A = \{1, 2, 3\}$

$R: A \to A$ s.t. $a >, b$, $a, b \in A$

$R = \{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (3, 1)\}$

Hasse Diagram :

Operation on Relation :

$$R_1 = \{(a, b), (b, a), (c, c), (c, b)\}$$

$$R_2 = \{(b, a), (b, b), (c, c)\}$$

$\Rightarrow$ $R_1\ U\ R_2 = \{(a, b), (b, a), (b, c), (c, b), (b, b), (c, c)\}$

$\Rightarrow$ $R_1\ n\ R_2 = \{(c, c)\}$

$\Rightarrow$ $R_1 \oplus R_2 = \{(a, b), (b, a), (b, c), (c, b), (b, b)\}$

$\Rightarrow$ $R_1 - R_2 = \{(a, b), (b, c), (c, b)\}$

$\Rightarrow$ $R_2 - R_1 = \{(b, a), (b, b)\}$

Complement of a Relation :

$$\overline{R} = (A \times A) - R$$

Eg. : $A = \{a, b, c\}$

$$R = \{(a, b), (b, c), (c, c), (b, a)\}$$

$$(A \times A) = \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,c)\}$$

$\Rightarrow$ $\overline{R} = (A \times A) - R$

$\Rightarrow$ $\overline{R} = \{(a,a),(a,c),(b,b),(c,a),(c,b)\}$

eg. $A = \{1, 2, 3\}$

$R : A \to A$ s.t. $a < b$

$A \times A = \{(1, 1,), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 3), (3, 2)\}$

$R = \{(1, 2), (1, 3), (2, 3)\}$

$\overline{R} = (A \times A) - R$

$\overline{R} = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)\}$

R Inverse : (R^{-1})

Inverse relation of R is, if $(a, b) \in R$

then $(b, a) \in R^{-1}$

Eg. :

$$R = \{(a,a),(b,c),(b,a),(a,b)\}$$

$$R^{-1} = \{(a, a), (c, b), (a, b), (b, a)\}$$

Q. **Closure of Relation :**

(1) Reflexive Closure :

$$R \cup Id$$

$\Rightarrow$ Id – Identity Relation

(2) Symmetric Closure :

$$R \cup R^{-1}$$

(3) Transitive closure :

if $O(A) = n$

$\Rightarrow$ $R \cup R^2 \cup R^3 \cup \ldots\ldots\ldots \cup R^n$ It is known as Transitive closure.

Composite Relation : Combination of multiple relation :

Eg. :– $R_1\ o\ R_2$

$A = \{a, b, c\}$

$R_1 = \{(a,a),(a,b),(c,b)\}$

$R_2 = \{(a,c),(b,b)\}$

$\Rightarrow$ $R_1\ o\ R_2 = \{(a, c), (a, b), (c, b)\}$

eg. $A = \{a,b,c,d\}$

$R_1 = \{(a,a),(a,c),(b,d),(b,c)\}$

$R_2 = \{(b,c),(a,d),(b,d)\}$

$\Rightarrow$ $R_1\ o\ R_2 = \{(a, d)\}$

$\Rightarrow$ $R_2\ o\ R_1 = \{\}$

(3) $A = \{p,q,r\}$

$R_1 = \{(p,p),(p,q),(p,r),(q,p)\}$

$R_2 = \{(p,r),(p,q),(q,r),(r,p)\}$

$\Rightarrow \quad R_1 \, o \, R_2 = \{(p,q),(p,r),(p,p),(q,r),(q,q)\}$

$\Rightarrow \quad R_2 \, o \, R_1 = \{(x,p),(p,p),(r,q),(r,r)\}$

Q. $A = \{+, -, *\}$

$R : A \to A$

$R = \{(+,+),(-,+),(+,*)\}$ find transitive closure of R.

Sol. : $R \cup R^2 \cup R^3$

$\Rightarrow R^2 = R \, o \, R$

$R = \{(+,+),(-,+),(+,*)\}$

$R = \{(+,+),(-,+),(+,*)\}$

$R^2 = \{(+,+),(+,*),(-,+),(-,*)\}$

$\Rightarrow R^3 = R^2 o R$

$R^2 = \{(+,+),(+,*),(-,+),(-,*)\}$

$R = \{(+,+),(-,+),(+,*)\}$

$R^3 = \{+,+\},(+,*),(-,+),(-,*)$

$\Rightarrow \quad R \cup R^2 \cup R^3 = \{(+,+),(+,*),(-,+),(-,*)\}$

Theorem :

The intesection of two equivalent relation is also an equivalent relation.

Proof : Let R and S are two equivalent relation. So 'R' be reflexive symmetric and transitive and also 'S' be reflexive, symmetric and transitive.

We have to prove that '$R \cap S$' also be an equivalent relation. For this we have to prove that '$R \cap S$' be reflexive, symmetric and transitive.

Reflexive :

$\because$ R is reflexive

So, $\quad \forall a \in A$

$(a, a) \in R$

$\because$ S is reflexive

So, $\quad \forall a \in A$

$(a, a) \in S$

So $\quad \forall a \in A$

$(a,a) \in R \cap S$

$\therefore$ $R \cap S$ is reflexive.

Symmetric :

Let $\forall a,b \in R \cap S$

If $\quad (a, b) \in R$

then $\quad (a, b) \in S$

$(b,a) \in R$ ($\because$ R is symmetric)

$(b, a) \in S$ ($\because$ S is symmetric)

$(b,a) \in R \cap S$

So, $R \cap S$ is Symmetric

Transitive :

$\forall a, b, c, \in A$

Let $\quad (a, b) \in R \cap S$ and $(b, c) \in R \cap S$

$(a, b) \in R$ and $(b, c) \in R$

$(a, b) \in S \qquad (b, c) \in S$

$\because \quad (a,b) \in R$ and $(b, c) \in R$

$\Rightarrow \quad (a, b) \in R$ ($\because$ R is transitive)

$\because \quad (a, b) \in S$ and $(b, c) \in S$

$\Rightarrow \quad (a, c) \in S$ [$\because$ S is transitive]

So, $\quad (a, c) \in R \cap S$

So, $R \cap S$ is transitive

Hence, $R \cap S$ is Equivalent Relation.

Theorem 2 :

Prove that R^{-1} of an equivalent relation is also an equivalent realtion.

Proof :

Let R be an equivalent relation i.e. R is reflexive, symmetric and transitive.

Now, we have to prove that R^{-1} is also an equivalent relation.

Reflexive :

$\because$ R is reflexive

$\Rightarrow$ $(a, a) \in R$

If $(a, a) \in R$

then $(a, a) \in R^{-1}$

So, R^{-1} is reflexive

Symmetric :

Let $(a, b) \in R^{-1}$

then $(b, a) \in R$

Now, R is symmetric

So, $(a, b) \in R$

$\Rightarrow$ $(b, a) \in R^{-1}$

So, R^{-1} is symmetric.

Transitive :

Let $(a, b) \in R^{-1}$ and $(b, c) \in R^{-1}$

then $(b, a) \in R$ and $(c, b) \in R$

Now, R is transitive

So, $(c, a) \in R$

$\Rightarrow$ $(a, c) \in R^{-1}$

So, R^{-1} is transitive

Hence, R^{-1} is an equivalent relation. (Proved)

Theorem 3 : Proof that $a = b$ (Mod k) is Equivalent Realtion.

Proof : $aRb \Rightarrow a = b(\text{Mod } K)$

$\Rightarrow$ $(a-b)\%\, K = 0$

Reflexive :

$a \equiv a(\text{Mod } K)$

$\Rightarrow$ $(a-a)\%K = 0$

$\Rightarrow$ $(a, a) \in R$

It is reflexive

Symetric :

$\Rightarrow$ $a \equiv b$ (Mod K)

$\Rightarrow$ $(a-b)\%\, K = 0$

$\Rightarrow$ $(b-a)\%\, K = -0$

$\Rightarrow$ $(b-a)\%\, K = 0$

$\Rightarrow$ $b \equiv a$ (Mod K)

$\Rightarrow$ $(b, a) \in R$

It is symmetric

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $a \equiv b(\text{Mod } K)$ and $b \equiv c$ (Mod K) ... (1)

$\Rightarrow$ $(a-b)\%K = 0$ and $(b-c)\text{Mod } K = 0$ ... (2)

$\Rightarrow$ Adding (1) + (2)

$[(a-b)+(b-c)]\%\, K = 0$

$\Rightarrow$ $(a-c)\%\, K = 0$

$\Rightarrow$ $a \equiv c(\text{Mod } K)$

$\Rightarrow \quad (a, c) \in R$

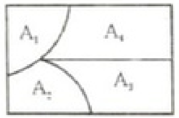
Hence, It is transitive,

So. $a \equiv b(\text{Mod } K)$ is an Equivalent Relation.

(Hence Proved)

* **Partition of set :** Any equivalent relation $R(A \to A)$ do partitioning of the set A in multiple disjoint equivalent classes.

Say $A_1, A_2 \ldots\ldots A_n$, these classes are mutually exclusive.



i.e. $[A_1] \cap [A_2] \cap \ldots\ldots\ldots \cap [A_n] = \phi$

and $[A_1] \cup [A_2] \cup \ldots\ldots\ldots \cup [A_n] = A$

$[a] = \{x : (a, x) \in R\}$

It is represauted by A/R, where set A is partitioned by equivalent relation R.

Eg :-

$A = \{1, 2, 3, 4\}$

$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, (3,1)\}$

$[1] = \{1, 2, 3\}$

$[4] = \{4\}$

Eg :- $a \equiv b(\text{mod } 4)$ defined on Set of intergess

$A \to \{\infty \ldots\ldots\ldots\ldots -2, -1, 0, 1, 2, 3, \ldots\ldots\ldots\ldots\}$

$[0] = \{\ldots\ldots\ldots\ldots -12, -8, -4, 0, 4, 8, 12, \ldots\ldots\ldots\ldots\}$

$[1] = \{\ldots\ldots\ldots\ldots -11, -7, -3, 1, 5, 0, 13 \ldots\ldots\ldots\}$

$[2] = \{\ldots\ldots\ldots\ldots -10, -6, -2, 2, 6, 10, 14 \ldots\ldots\ldots\ldots\}$

$[3] = \{\ldots\ldots\ldots\ldots -9, -5, -1, 3, 7, 11, 15 \ldots\ldots\ldots\}$

Theorem 4 : (Duality Priniciple)

Prove that dual of a POSET is also POSET

Proof : POSET– partially order set

set $(A, \leq)$ Here $A \to$ set, $\leq_R \to$ P.O.R.

So, POSET is a structure which have 2 elements, one is set and other is partially order relation

Eg. :

(z, 1)

Dual of POSET $(A, \leq_R)$ is $(A, R\geq)$:

$a \leq_R b$

$a R\geq b$

Dual

Let $(A, \leq_R)$ is a POSET, So $\leq_R$ is reflexive, antisymmetric and transitive. We have to prove that Dual of POSET given is also poset.

For this we have to prove that $R\geq$ is reflxive, Antisymmetric transitive :

Reflexive :

$\forall a \in A$

$a \leq_R a \qquad (\because \leq_R \text{ is reflexive})$

$\Rightarrow \quad a R\geq a$

Transitive :

Let $a \leq_R b$ and $b \leq_R c$

$\Rightarrow \quad a \leq_R b$ and $b \leq_R c$

$\Rightarrow \quad a \leq_R c \quad (\because \leq_R \text{ transitive})$

$\Rightarrow \quad a \geq_R c$

Anti-symmetric :

Let $\quad a \leq_R b$

$\Rightarrow \quad a \leq_R b$

$\Rightarrow \quad b \leq_R a \qquad (\text{if } a = b)$

$\Rightarrow \qquad b \geq_R a \qquad (\text{if } a = b)$

So, $\quad \geq_R$ is POR

Hence, Dual of POSET $(A, \leq)$ is POSET.

(Hence Proved)

Russel Paradox :

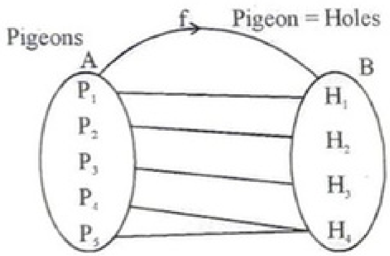
Every collection is not set only well defined collection is set.

Eg : Barber paradox

There is a barber in a village. He shave only there who don't Shave themselves. This collection is not well defined. Since Barber's condition is not clear. he may be part of this collection or not be part of this collection. So, this collection is not set.

Pigeon-Hole Principle : If pigeons are more than pigeon hole then these exist atleast one such a pigeon hole that contains more than one pigeon.

Eg :–



Eg :–

(1) In a group of 13-members atleast 2 members born in same mouth.

(2) A patient have to take 32 tables in a month then these exist atleast one day in which he take more than one tablet.

(3) 7 meabers, 5 chairs then there exist one such a chair that contain more than one member.

In function form :

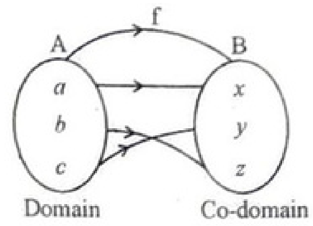
Let A is set of pigeons and B is set of pigeon-holes and |A| > |B|

then $\exists x_1, x_2 \in A$ s.t. $x_1 \neq x_2$

but $\qquad f(x_1) = f(x_2)$ (many-to-one function)

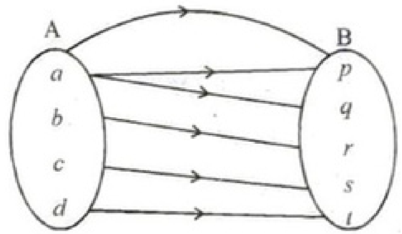
Function : A relation is known as function if every element of domain have an unique image in codomain.

Eg. (1)



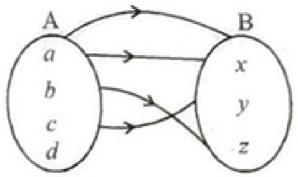
$f(a) = x$
$f(b) = y$
$f(c) = z$

eg.



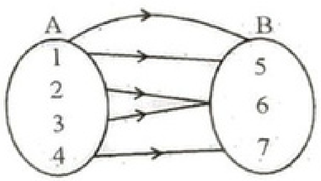
(Not function)

eg.



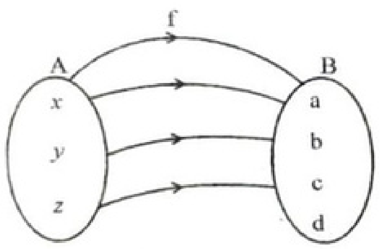
(Not Function)

eg.

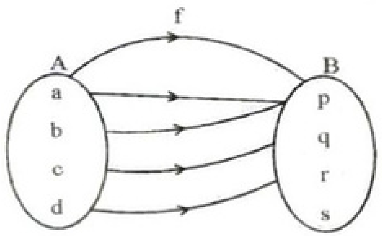


$f(1) = 5$
$f(2)$ and $f(3) = 6$
$f(4) = 7$

eg.



$f(x) = a$
$f(y) = b$
$f(z) = c$



$f(a)$ & $f(b) = p$
$f(c) = q$
$f(d) = r$

Domain $\{a, b, c, d\}$

Condomain : $\{P, q, r, s\}$

Range : $\{p, q, r\}$

Note:- Every function is also Relation but not vice-versa.

Types of functions :

(1) One-to-one/Many-to-one

(2) Into on to

one-one – **Injection**

onto – surjection

one-one and onto – **Bijection**

one-one:-

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

then one-one

Onto:-

if Range = Co-domain

eg. $f(x) = 2x$

$f : N \to N$

one-to-one and onto function.

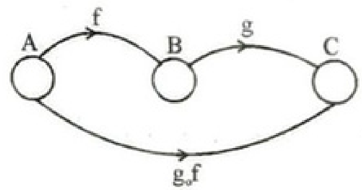
eg. $f(x) = 2x^2$

$f : z \to z$

may-to-one and into function

Composition of Functions :

$f(x)$	$g(x)$
$fo_g(x)$	$go_f(x)$
$f(g(x))$	$g[f(x)]$



Eg. : $f(x) = 3x + 4$

$g(x) = 2x^2$

$fo_g(x) = f[g(x)]$

$= f[2x^2]$

$= 3(2x^2) + 4$

$= 6x^2 + 4$

$go_f(x) = g[f(x)]$

$= g[3x+4]$

$= 2(3x+4)^2$

Eg. : $f(x) = x^2 | 4$

$g(x) = x/3$

$fo_g(x) = f[g(x)]$

$= f[x/2]$

$= (x/2)^2 | 4$

$= \frac{x^2}{4 \times 4}$

$= \frac{x^2}{16}$

$go_f(x) = g[f(x)]$

$= g(x^2/4)$

$= \frac{x^2}{4x^2}$

$= \frac{x^2}{8}$

Eg. : $f(x) = 2x+5$

$g(x) = 3x^2$

$h(x) = 4x-5$

$fo(goh)(x) = fo[g[h(x)]]$

$= fo[g[4x-5]]$

$= fo[3(4x-5)^2]$

$= 2(3(4x-5)^2)+5$

$= 2\left(3(16x^2+25-40x)\right)+5$

$= 96x^2+150-240x+5$

$= 96x^2-240x+155$

Eg. : $f(x) = 3x+4$

$g(x) = 2x^2$

$fof(x) = f[f(x)]$

$= f(3x+4)$

$= 3(3x+4)+4$

$= 9x+12+4$

$= 9x+12+4 = 9x+16$

$gog(x) = g[g(x)]$

$= g[2x^2]$

$= 2(2x^2)^2$

$= 2(4x^4)$

$= 8x^4$

Inverse function :

Inverse in possible only for one-one and onto function i.e. Bijective function.

Eg: $f(x) = 2x$

$f^{-1}(x) = ?$

Sol. : $f(x) = y$

$\Rightarrow \quad y = 2x$

$\Rightarrow \quad x = \frac{y}{2}$

$\Rightarrow \quad f^{-1}(x) = \frac{y}{2}$

$\Rightarrow \quad f^{-1}(x) = x/2$

Eg. :

$f(x) = 3x - 4$

$\Rightarrow \quad f(x) = y$

$\Rightarrow \quad y = 3x - 4$

$\Rightarrow \quad \frac{y+4}{3} = x$

$\Rightarrow \quad f^{-1}(x) = \frac{x+4}{3}$

Questions

Very Short type:-

1. If O(P) = 3, O(Q)=4, O(R) = 2
   then find O(P x Q x R)
2. Find the number of subsets of the set *{2,3,5}*
3. n(A)= 70, n(B) = 32, $n(A \cup B) = 22$
   find $n(A \cup B)$
4. $A = \{x \mid X < 4;\ x \in N\}$
   $B = \{x \mid x \le 6;\ x \in w\}$
   find B-A

5. Find the number of subsets of set $A = \{x \mid 4 < x \le 7, x \in N\}$
6. $g(x) = x(x-1)$. Find $g\left(-\frac{1}{2}\right)$
7. Define Russel paradox.
8. What is multiset.
9. What is transtive relation.
10. f(x) = 9x+6 find f^{-1} (x)

Short type:-

1. $A = \{1,2,3,4,5\}$, $B = \{2,4\}$, $C = \{1,3,5,\}$
   find $(A - C) \times B$
2. A survey shows that 68% of women like apples. 74% of women like bananas.what percentae like both apples & bananas.
3. In a group of 23 children 10 drink tea but nat coffee and 15 like tea. Find the number of children drinking coffee but not tea.
4. In a class of 30 students, 20 like maths, i8 like science & 12 like both. Final the number of student who like no subject.
5. $f(x) = 2x + 3$ find f log(x) and gof(1)
6. f(x)=2x+3
   calculate f(2x) –2f(x) + 3
7. f(x) = 2x+h
   find f(x+h) – 2f(x)
8. $A = \{\pm 2, \pm 3\}$, $B = \{1, 4, 9\}$
   $f : A \to B$ S.t. $f(x)=x^2$
   Find nature of f
9. $A = \{1, 4, 5, 6\}$
   R: $A \to A$ S.t.
   $a > b - 6$, $(a, b \notin R)$
   Represent this seltion in matrix form and find its type.

10. $A = \{x \mid -2 \le x \le 2, x \in z\}$

$B = \{x \mid x \le 10, x \in z\}$

$f : A \to B$ s.t. $f(x) = x^2 + 3$

find natural of f

Long type:-

1. Find number of natural numbers upto 500 which are divisible by 2, 3, 5.
2. Draw venn Diagrams for:-

   (a) $A-(A \cap B)$

   (b) $\overline{A} \oplus (A-B)$

   (c) $(A-B) \cup (\overline{A} \cup \overline{B})$

   (d) $(\overline{A \cup B})-(A \cup B)$
3. Prove $(A \cup B)' = A' \cup B'$
4. Prove $A-(B \cap C) = (A-B) \cup (A-C)$
5. Prove $A \times (B \cup C) = (A \times B) \cup (A \times C)$
6. $A = \{1, 2, 3, 4, 5, 6\}$

   $R : A \to A$ s.t $a = b^k$ $(a, b, k \in A)$

   Find type of Reltion R Represent this into graphical and matrics form.
7. State and prov distributive and Associative laws for stes.

❑❑❑