Unit-II

Magnetic field due to a bar magnet, Biot Savrt's law, magnetic field due to a current carrying coil, Force between two parallel currents, Magnetic field inside solenoid and toroid, magnetic flux, Faraday's law of electromagnetic induction, magnetic properties of matter, (diamagnetic, paramagnetic, ferromagnetic and ferromagnetic materials), inductance, energy stored in an inductor, LR circuits.

2. Magnetism 63-113

Chapter

2 Magnetism

1. Introduction

If a magnetic needle is suspended freely in a filed and a torque acts on it such that the needle comes to rest after rotating in a definite direction, then the force field acting on the magnetic needle is called **magnetic field.**

Our earth also produces a magnetic field as a result if a magnetic needle is suspended freely it always comes to rest in the north-south direction.

The magnetic field intensity and its direction at different points in a magnetic field can be represented by lines of force. These lines of force can be obtained either by tapping the iron filings spread in the field or by marking the rest position of a magnetic needle at different points in the plane of magnetic field.

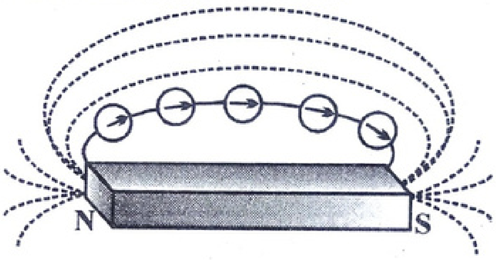

Fig. (2.1) : Magnet

The line of force in a magnetic field is that imaginary continuous line, the tangent to which at every point provides the direction of magnetic field at that point. Only one direction of field is possible at one place. So no two lines of force intersect each other. Magnetic lines of force are in the form of closed curves. In external region they start from north pole of the magnetic and go to south pole, whereas inside the magnet they are from south pole to north pole. Where the force field is strong the number density of lines of force is greater. In weak fields the lines of force are at large distances apart. The number of lines of force crossing certain area is called flux.

The number of lines of force passing through a unit area placed perpendicular to the field is called magnetic flux density of magnetic induction. Magnetic field intensity can be represented by this flux density. If the magnetic field intensity at a certain point is B, then the magnetic flux passing through an area element $\vec{da}$ is $\vec{B} \cdot \vec{da}$. In vector form the direction of area element is outward normal to that area. Hence

$$\text{Magnetic flux } d\phi = \vec{B}.\vec{da}$$

If the direction of vector area $\vec{da}$ is in the direction of $\vec{B}$, i.e., the plane of area is perpendicular to the magnetic field, then

$$d\phi = B\, da$$

From the definition of flux

$$B = \frac{d\phi}{da} \text{ Wb/m}^2 \text{ or tesla}$$

In uniform magnetic field the magnetic lines of force are parallel and equidistant from each other. This shows that the direction and intensity of the magnetic field are same at all points.

If we draw magnetic lines of force of a bar magnet, then it is found that all lines of force appear to be concentrated at the ends of the bar magnet. In fact the lines at the ends of the magnet are very near each other because the intensity is quite large there. [Fig. (2.2)]. In earlier day it was assumed that magnet has two poles like an electric dipole. On suspending a magnet the end which points towards north of the earth is called north pole and that which points towards south of the earth is called south pole. Magnetic poles can not be isolated. If a magnet is broken into pieces, then each piece will contain both poles, that is every piece shall be a complete magnet. But if we take a thin and long magnet, then its poles can be considered approximately isolated. Using this hypothesis Coulomb gave a law which states that if two magnetic poles are at a distance of d apart then they experience a force between them which is inversely proportional to the square of the distance and directly proportional to the product of the pole strengths.

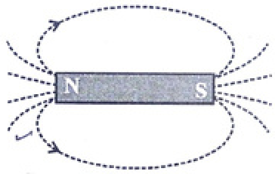

Fig. : 2.2 : Bar magnet

Thus

$$F \propto m_1 m_2$$

$$\propto \frac{1}{d^2}$$

or

$$F = K' \frac{m_1 m_2}{d^2}$$

where K' is a constant whose value in rationalised MKS system is 10^{-7} weber/ampere-metre.

The unit of pole strength m is ampere-metre.

$$\therefore \quad F = 10^{-7} \frac{m_1 m_2}{d^2} \text{ newton}$$

The intensity of magnetic field at any point which is also called magnetic induction B is defined as force per unit north pole acting on any magnetic pole placed at that point.

If a magnetic pole of strength m' experiences of a force F in a field then

$$B = \frac{F}{m'}$$

To obtain the magnetic field intensity at a point P due to a magnetic pole m, we place another pole of strength m' at that point and find the force acting on it.

By Coulomb's law the force is given by

$$\vec{F} = K' \frac{mm'}{r^2} \hat{r}$$

where r is the distance of the point from the pole m.

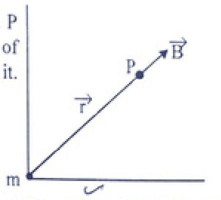

Fig. : 2.3 : Intensity of magnetic field at point P

$\therefore$ Intensity of magnetic field at P

$$\vec{B} = \frac{\vec{F}}{m'} = K' \frac{mm'}{r^2} \times \frac{1}{m'} \hat{r}$$

$$= K' \frac{m}{r^2} \hat{r}$$

In rationalised MKS units it is convenient to write

$$K' = \frac{\mu_0}{4\pi}$$

when the poles are situated in free space. μ_0 is called permeability of free space.

$$\mu_0 = 4\pi K'$$
$$= 4\pi \times 10^{-7}$$
$$= 12.56 \times 10^{-7} \text{ Henry/metre}$$

Hence in free space the magnetic field intensity or magnetic induction at a point due to a magnetic pole is

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{m}{r^2} \hat{r} \text{ weber/m}^2 \text{ or tesla.}$$

2. Magnetic Field due to a Bar Magnet

A magnetic can be considered as magnetic dipole. If the pole strength is m then the pole strength of the north pole will be +m and the pole strength of south pole will be –m. The distance between the poles of the magnet is called its effective length and represented by $2l$. **The product of pole strength m and the distance between the poles $2l$, is called magnetic moment of the magnet. Thus magnetic moment $M = m(2l) = 2ml$.** The unit of magnetic moment is ampere-metre^2: The line joining the poles of a magnet is called the axis of the magnet while the line passing through the centre and perpendicular to the axis is called its equatorial line. Depending on the position of the point with respect to the magnet the determination of magnetic field can be treated in three parts.

(i) Point on the axis of the magnet (End-on position) :

Consider a point P on the axis of a bar magnet at a distance r from its centre. The distance of the point P from the N-pole will be $(r - l)$ while from S-pole it will be $(r + l)$. The magnetic field intensity at P due to N-pole will be $\frac{\mu_0}{4\pi}\frac{m}{(r-l)^2}$ and due to S-pole it will be $\frac{\mu_0}{4\pi}\frac{m}{(r+l)^2}$.

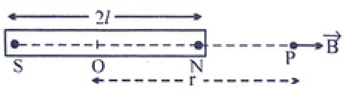

Fig. (2.4) : Axis of the Bar magnet

∴ Resultant magnetic field intensity at P is

$$B = \frac{\mu_0}{4\pi}\left[\frac{m}{(r-l)^2} - \frac{m}{(r+l)^2}\right] \text{ along the axis.}$$

$$= \frac{\mu_0}{4\pi}\frac{m(r+l)^2 - m(r-l)^2}{(r^2-l^2)^2}$$

$$= \frac{\mu_0}{4\pi}\frac{4mlr}{(r^2-l^2)^2} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2}$$

where $M = 2ml$ (magnetic moment of the magnet).

For a short magnet $r >> l$, so l^2 will be negligible compared to r^2.

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{2Mr}{r^4} = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$

(ii) Point on the equatorial line (Broad-side-on position) :

Consider a point P on the equatorial line of a magnet at a distance r from its centre. The distance of P from both the poles will be the same, equal to $(r^2 + l^2)^{1/2}$

The intensity of magnetic field due to N-pole will be

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)} \text{ along PQ.}$$

The intensity of magnetic field due to S-pole will be

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)} \text{ along PR.}$$

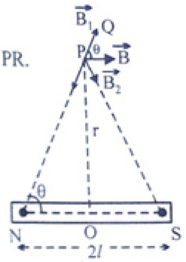

Fig. (2.5) : Equatorial line of a magnet

If we resolve $\vec{B}_1$ and $\vec{B}_2$, parallel to the axis and perpendicular to the axis, the components perpendicular to the axis being equal and opposite will cancel each other while the components parallel to the axis being in the some direction will get added.

∴ Resultant magnetic field at P will be in a direction parallel to the axis and will be given be

$$B = B_1 \cos\theta + B_2 \cos\theta$$

$$= \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)}\cos\theta + \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)}\cos\theta$$

$$= \frac{\mu_0}{4\pi}\frac{2m}{(r^2+l^2)}\frac{l}{(r^2+l^2)^{1/2}} = \frac{\mu_0}{4\pi}\frac{M}{(r^2+l^2)^{3/2}}$$

For a small magnet $l << r$, so that

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

(iii) Point lying on a line making an angle θ with the axis of the magnet :

For simplicity the magnet is considered small so that $l << r$. For this position the magnetic moment can be resolved into two components, M cos θ along OP and M sin θ perpendicular to OP.

For the component M cos θ the point P will be in end-on position so the intensity of magnetic field at P will be

$$B_1 = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3} \text{ along OP.}$$

For the component M sin θ the point P will be in broad side-on position so the intensity of magnetic field at P will be

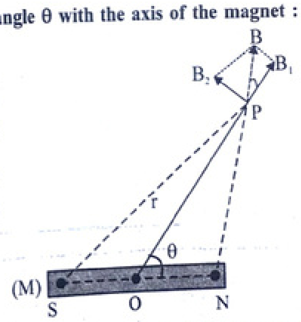

Fig. (2.6) : Point lying on a line making an angle θ with the axes of the magnet

$$B_2 = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$, perpendicular to OP.

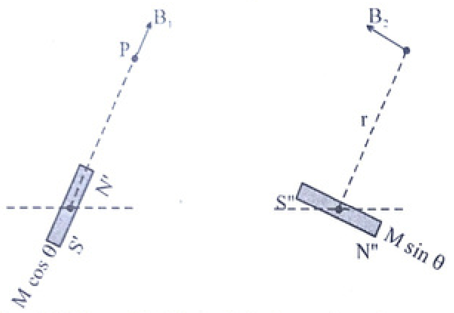

Fig. : 2.7 : Board side-on Position of the intensity of magnetic field at P

Resultant intensity of magnetic field

$$B = (B_1^2 + B_2^2)^{1/2}$$

$$= \frac{\mu_0 M}{4\pi r^3}[1 + 3\cos^2\theta]^{1/2}$$

If the field $\vec{B}$ makes an angle α with $\vec{r}$ then

$$\tan\alpha = \frac{B_2}{B_1} = \frac{1}{2}\tan\theta$$

3. Biot-Savart's Law

When current flows through a conductor a magnetic field is produced around the conductor. Biot and Savart proposed a law for determining the magnetic field intensity at a certain point on the basis of these experiments. This law is called Biot-Savart's law. According to this law the magnetic field intensity δB at a certain point P due to a current carrying element δl of the conductor is

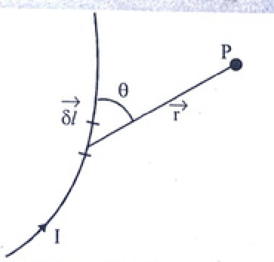

Fig. (2.8) : Biot-Savart's law

(i) directly proportional to the current I,

(ii) directly proportional to the length δl of the current element,

(iii) directly proportional to sin θ, where θ is the angle between the position vector $\vec{r}$ of the point of observation with respect to the element and the element $\vec{\delta l}$ and

(iv) inversely proportional to the square of the distance r of the observation point from the element, i.e.,

$$\delta B \propto \frac{I\delta l\sin\theta}{r^2}$$

In free space or non-magnetic medium.

$$\delta B = \left(\frac{\mu_0}{4\pi}\right)\frac{I\delta l\sin\theta}{r^2} \text{ weber/m}^2 \text{ or tesla.}$$

where μ_0 is called magnetic permeability of free space and its value is $4\pi \times 10^{-7}$ weber/A-m or henry/m.

For other media μ is used in place of μ_0 for the permeability of that medium. The ratio of μ and μ_0, i.e., $\mu_r = \frac{\mu}{\mu_0}$ is called relative permeability of the medium.

The direction of magnetic field is normal to the plane containing the element $\vec{\delta l}$ and the position vector $\vec{r}$ and its sense can be determined by Maxwell's right hand screw rule or right hand palm rule.

Biot-Savart' law in vector form is

$$\vec{\delta B} = \frac{\mu_0}{4\pi}\frac{I\vec{\delta l}\times\hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\vec{\delta l}\times\vec{r}}{r^3}$$

Case 1 : If θ = 0 i.e, the position vector of the observation point is parallel to the direction of flow of current then sin θ = 0 and δB = 0, which is its minimum value.

Case 2 : If $\theta = \frac{\pi}{2}$ i.e., the position vector of the observation point is perpendicular to the direction of flow of current, then sin θ = 1 and

$$\delta B = \frac{\mu_0}{4\pi}\frac{I\delta l}{r^2},$$

which is its maximum value.

Direction of magnetic field

The direction of magnetic field is determined with the help of the following simple laws :

(a) Maxwell's cork-screw rule : According to this law if a right hand screw is rotated in such a way that it moves forward in the direction of current in the conductor, then the direction of rotation of the screw will show the direction of lines of force, Fig. (2.9)

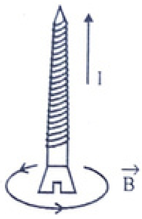

Fig. (2.9) :Maxwell's cork-screw rule

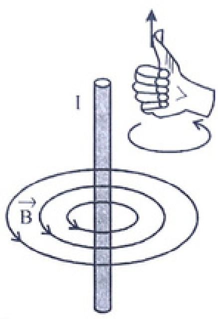

Fig. (2.10) : Right hand palm rule

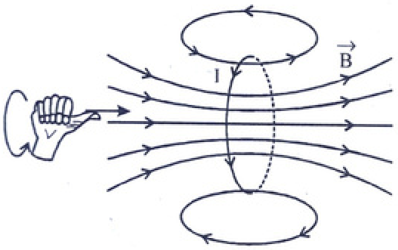

Fig. (2.11) : Right hand Palm rule for circular current

(b) Right hand palm rule : According to this rule if a current carrying conductor is held in the right hand such that the thumb represents the direction of magnetic lines of force Fig. (2.10)

(c) Right hand palm rule for circular currents : According to this rule if the direction of current in a circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb Fig. (2.11).

4. Magnetic Field due to a Circular Coil Carrying Current

(i) Observation point is at the centre of coil : When the observation point is at the centre of a coil, the distance of observation point, from each element of

the coil is equal to the radius 'a'. In addition the position vector $\vec{r}$ makes an angle of 90° with each element. Hence sin θ = sin 90° = 1. Thus from Biot-Savart's law the magnetic induction due to a current carrying element δl at the point P.

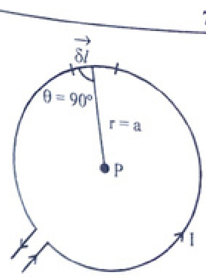

Fig. (2.12) : Magnetic Field due to a circular coil

$$\delta B = \frac{\mu_0}{4\pi}\frac{I\delta l}{a^2}$$

The direction of magnetic field due to all elements is same (normal to the plane of coil). As a result total magnetic induction at the centre of the coil is

$$B = \Sigma\, \delta B$$

$$= \frac{\mu_0}{4\pi}\frac{I}{a^2}\Sigma \delta l$$

$$= \frac{\mu_0 I}{4\pi a^2}(2\pi a) = \frac{\mu_0 I}{2a}$$

In the coil has n turns, then

$$\Sigma \delta l = 2\pi a n$$

$$\therefore \quad B = \frac{\mu_0 n I}{2a} \text{ weber/m}^2 \text{ or tesla.}$$

(ii) Observation point is at a distance x from the centre and on the axis of a coil : In this case the observation point P is at the same distance r from each element of the coil, $r = \sqrt{a^2 + x^2}$. Further the angle θ between the current element $\vec{\delta l}$ and the position vector $\vec{r}$ is 90° as the position vector $\vec{r}$ is in the plane of paper and the element $\vec{\delta l}$ is perpendicular to the plane of paper. Hence sin θ = sin 90° = 1.

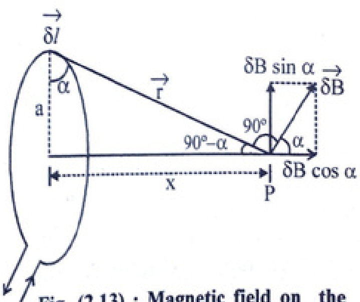

Fig. (2.13) : Magnetic field on the axis of a coil

According to Biot-Savart's law the magnetic induction due to a current element is

$$\delta B = \frac{\mu_0}{4\pi} \frac{I\delta l}{a^2}$$

The direction of $\vec{\delta B}$ is normal to the plane containing the position vector $\vec{r}$ and the current element $\vec{\delta l}$. Suppose $\vec{\delta B}$ makes an angle α with the axis of the coil. $\vec{\delta B}$ can be resolved in components along the axis and perpendicular to the axis. The components $\delta B \cos\alpha$ of magnetic induction $\vec{\delta B}$ due to different elements along the axis are in the same direction but the perpendicular components $\delta B \sin\alpha$ are symmetrically distributed in different directions around the axis. Thus the magnetic induction due to the coil at the observation point $\vec{B} = \Sigma\vec{\delta B} = \Sigma \delta B \cos\alpha$ along the axis as $\Sigma\delta B \sin\alpha$ normal to the axis is equal to zero.

Thus
$$B = \Sigma \frac{\mu_0}{4\pi} \frac{I\delta l \cos\alpha}{r^2} = \frac{\mu_0}{4\pi} \frac{I\cos\alpha}{r^2} \Sigma\delta l$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r^2} \frac{a}{r} (2\pi n a)$$

$$= \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{r^3}$$

$$= \frac{\mu_0 n I a^2}{2(a^2 + x^2)^{3/2}} \text{ tesla (along the axis).}$$

If a graph is plotted between the magnetic induction B against the distance x from the centre then the curve as shown in Fig. (2.14) is obtained. In this curve points of inflection are obtained at the positions $x = \pm a/2$. The points of inflexion are those points where the curvature becomes zero and the direction of curvature changes sign. The variation of B with x near these points is linear. The distance between these two points of inflexion is equal to the radius of the coil.

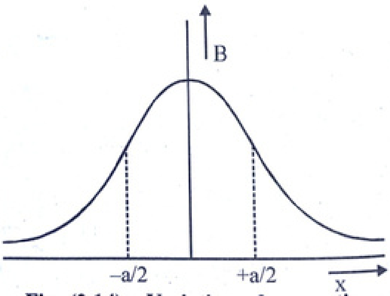

Fig. (2.14) : Variation of magnetic induction B with the distance x from the centre

5. Magnetic Induction due to a Long Current Carrying Wire

Consider a long straight wire in which a current I is flowing. The wire is in the plane of paper. The length of the wire is very large as compared to the distance r of the point at which magnetic induction is to be determined, so that the wire can be treated as infinitely long.

Due to flow of current in the conductor a magnetic field is produced all around it and in this case the magnetic lines of force will be in the form of concentric circles around the wire.

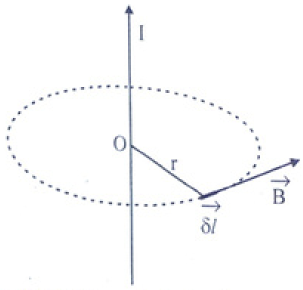

Fig. (2.15) Magnetic Induction to a long current carrying wire

The magnetic induction at a distance r from the wire is given by :

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{4\pi}\left(\frac{2I}{r}\right)$$

Thus, **the magnitude of magnetic induction B due to a long current carrying wire at a point near it is (i) directly proportional to the current I and (ii) inversely proportional to the distance r.**

6. Force between Two Current Carrying Parallel Conductors

In the figure two parallel conductors are shown. The distance between them is d. Let i_a and i_b be the currents flowing in these conductors. When currents flowing in these conductors are in the same direction, they attract each other. When currents flowing in them are in opposite directions, they repel each other. The magnetic field produced by the current i_a flowing in the wire 'a' at a distance d is

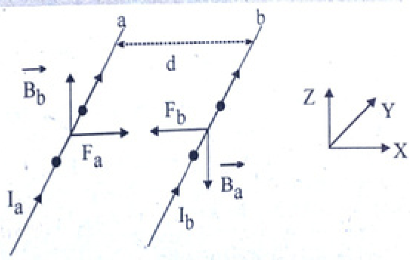

Fig. (2.16) : Two parallel conductors

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

According to the Fig. (2.16) if the current is in y-direction and position vector of a point is in x-direction, then the magnetic field will be (–z) direction.

The wire b carrying a current i_b is situated in the magnetic field B_a produced by the wire a. Consider an element of length δl of wire b. The magnitude of force on the element of wire b will be

$$F_b = i_b \, \delta l \, B_a$$

because current element $\vec{\delta l}$ and $\vec{B}_a$ are perpendicular to each other.

$$\therefore \quad F_b = \frac{\mu_0 i_a i_b \delta l}{2\pi d}$$

The direction of F_b will be towards the conductor a.

Thus the force per unit length of wire b

$$\frac{F_b}{\delta l} = \frac{\mu_0 i_a i_b}{2\pi d}$$

Similarly the force per unit length of wire 'a' due to the magnetic field produced by the wire b will be

$$\frac{F_a}{\delta l} = \frac{\mu_0 i_a i_b}{2\pi d}$$

This force will be in (+x) direction, i.e. towards the conductor b.

7. Definition of Ampere

If same amount of current $i_a = i_b = 1$ A is flowing in the wires and d the distance between the wires is 1m, then

$$\frac{F_a}{\delta l} = \frac{F_b}{\delta l} = \frac{\mu_0}{2\pi} = 2 \times 10^{-7} \text{ N/m}$$

Thus if **two infinitely long thin conductors are placed parallel to each other at a distance 1m apart each carrying same current in the same direction, and a force of attraction of 2×10^{-7} N/m acts between them, then the current flowing in each wire will be one ampere. This definition has been accepted as an International Standard.**

8. Magnetic Induction Inside a Long Straight Solenoid

A solenoid consists of a hollow cylindrical tube on which electrically insulated wires are wound uniformly along its length. The diameter of the solenoid is kept

much smaller compared to its length and in a ideal solenoid the plane of each turn of the wire can be considered perpendicular to its length. When a current is passed through the solenoid the magnetic field near each wire is due to a straight current carrying wire and the lines of force are concentric circles, as shown in Fig. 2.17.

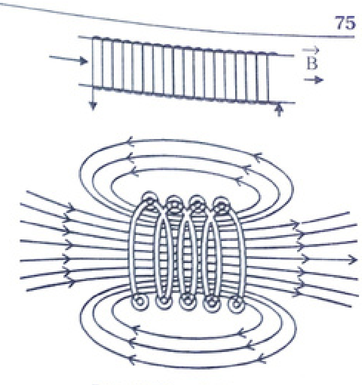

Fig. (2.17) : Solenoid

The magnetic field $\vec{B}$ at any point due to the solenoid is the vector sum of magnetic field intensities due to all the turns. **The resultant field inside the solenoid is thus uniform and along the axis.** At points outside the solenoid the field is negligible compared to the field inside. For a long solenoid at points sufficiently outside it the field can be treated as zero.

If the current flowing through the solenoid is I and the number of turns per unit length is n then inside the solenoid

$$B = \mu_0 n I$$
$$= \mu_0 \frac{NI}{L}$$

where N is total number of turns and L is the length of solenoid.

If the medium inside the solenoid has a permeability μ then $B = \mu n I$. $\frac{\mu}{\mu_0} = \mu_r$ is relative permeability.

9. Magnetic Induction in a Toroid

A toroid is an endless solenoid i.e., if a long solenoid is bent in a circular form and its ends are joined, it becomes a toroid. In a toroid electrically insulated wire is wound uniformly over a torus (circular ring) as shown in Fig. (2.18). The thickness of toroid is kept small compared to its radius and the number of turns is kept very large. When a current

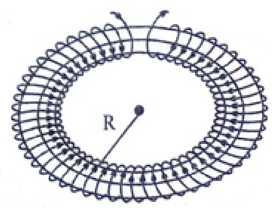

Fig. (2.18) : Toroid

I is passed through the toroid then at its center each turn of the toroid produces a magnetic field along its axis. Due to the uniform distribution of the turns the magnetic field at the centre of each turn is of the same magnitude and the magnetic lines of force inside the toroid are circular with centre at the centre of toroid. This magnetic field is only in the space covered by the turns. Outside the toroid the magnetic field is negligible (~ zero).

Let the current flowing through the toroid be I, N be the number of turns in it and R be its mean radius. Then inside the toroid the magnetic induction is

$$B = \mu_0 nI = \mu_0 \frac{N}{2\pi R} I$$

where $n = \frac{N}{2\pi R}$ is number of turns per unit length.

If the toroid is made on a ring a permeability μ, then $B = \mu nI$.

10. Magnetic Flux

The concept of magnetic lines of force was given by Faraday. Faraday tried to piturize the lines of force as stretched rubber bands. In present day physics the idea of lines of force is only used for the visualization and explanation of physical principles. For quantitative calculations field vectors are used. Through a close correlation between picturization and mathematical analysis the understanding of various principles becomes easier and more realistic. **In a magnetic field the tangent to a lines of force at a given point gives the direction of the magnetic field at that point and their density i.e., the number of lines of force passing through a unit area perpendicular to the field represents the intensity of the filed.** In a uniform magnetic field the lines of force are parallel and equidistant straight lines. Wherever the lines of force are closely spaced the intensity of field B is more and wherever these are at larger distances the intensity of field B is less.

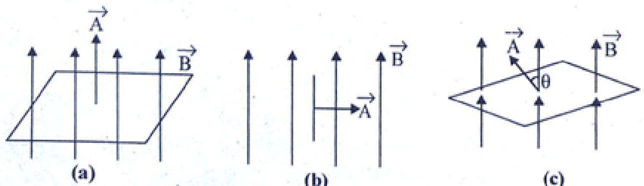

Fig. (2.19) : Magnetic flux

The flux, which is represented by ϕ, is a characteristic of a vector field. The number of lines of force passing through a given surface is called the flux through that surface. For simplicity if we imagine a plane surface of area A placed in a uniform magnetic field $\vec{B}$, the number of lines of force passing through this area i.e., magnetic flux ϕ will be maximum equal to BA when the area is perpendicular to the field Fig. (2.19a). If the area is parallel to the field then flux ϕ is zero, Fig. (2.19b). When the perpendicular to the area makes an angle θ with the direction of the field then the flux ϕ becomes BA cos θ, Fig. (2.19 c). A plane area can be represented by a vector with a direction which is outward normal to it (actually area is a scalar but can be represented as a vector). Thus in Fig. (2.19c) the angle θ is the angle between magnetic field vector $\vec{B}$ and area vector $\vec{A}$ so that the flux BA cos θ is equal to scalar product of $\vec{B}$ and $\vec{A}$. In general, for a plane surface $\phi = \vec{B}.\vec{A}$. In the situation given in Fig. (2.19a) $\theta = 0$ (normal to area in the direction of field $\vec{B}$) so that $\vec{B}\cdot\vec{A} = BA \cos 0 = BA$, while in the situation given in Fig. (2.19 b) $\theta = 90°$ (normal to area perpendicular to the field $\vec{B}$) therefore $\phi = BA \cos 90° = 0$.

If the magnetic field is not uniform and the given surface is not plane, then any element $\vec{d}A$ of area can be treated as plane and at the element the field can also be treated as uniform. Hence the flux emerging out of an area element will be

$$d\phi = \vec{B}.\vec{d}A,$$

and the total flux through a given surface S will be

$$\phi = \int_S \vec{B}.\vec{d}A$$

For a closed surface the area elements with outward normal direction are treated positive and area elements with inward normal direction are taken negative.

The magnetic lines of force are closed curves because free magnetic poles do not exist. Hence for a closed surface the number of lines of force which enter into it is equal to the number of lines of force which emerge out of it. As a result for a closed surface

$$\phi = \oint \vec{B}.\vec{d}A = 0.$$

This result is different from the result obtained for an electrostatic field because the source of electric field that is electric charge can freely exist.

For a plane surface perpendicular to a magnetic field

$$\phi = BA \text{ or } B = \frac{\phi}{A}$$

i.e., the **magnetic induction B can be defined as the magnetic flux passing**

per unit area of a plane surface perpendicular to the field. $B = \frac{\phi}{A}$ is also called flux density.

Units of magnetic flux : In M.K.S. system the unit of magnetic flux is 'weber' (Wb) and in C.G.S. system the unit is 'maxwell'.

$$1 \text{ weber} = 10^8 \text{ maxwell.}$$

The M.K.S. unit of flux density or magnetic induction B is weber/m^2 and is also called tesla (T).

$$1 \text{ tesla} = 1 \text{ weber/m}^2.$$

The C.G.S. unit of magnetic flux density is 'gauss'.

$$1 \text{ gauss} = 1 \text{ maxwell/cm}^2$$

$$1 \text{ tesla} = 1 \text{ weber/m}^2 = 10^4 \text{ gauss.}$$

11. Faraday's Experiments and Induced E.M.F.

In 1831 Michael Faraday in England and almost at the same time Joseph henry in America demonstrated that when a bar magnet is swiftly brought near a coil connected to a galvanometer, the galvanometer shows a deflection Fig. (2.20) Furthermore, the faster the magnet moves the greater is the deflection and when the movement is stopped the deflection becomes zero.

There is no cell or source of e.m.f in the circuit. However, the deflection in the galvanometer shows that a current has been produced and there must be an e.m.f. developed. When the magnet is moved away swiftly again a deflection is observed in the galvanometer, but now in opposite direction. If the magnet is reversed (end for end) the results are as before except that the directions of deflections are reversed. Thus, the motion of the magnet towards or away from the loop is the reason for the deflection in the galvanometer. Further experiments prove that it is the relative motion of the magnet and the loop which produces an e.m.f. in the loop.

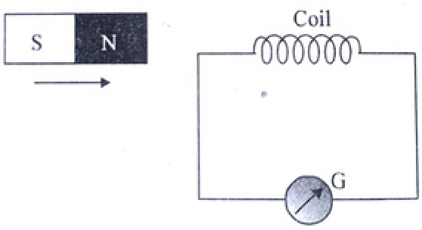

Fig. (2.20) : Faraday Experiment

This phenomenon was named by Faraday as **electromagnetic induction.** The e.m.f. that is produced in the coil is called **induced e.m.f.** and the current obtained is called **induced current.**

Regarding this phenomenon Faraday observed the following :

(i) Galvanometer has a deflection till the magnet is in motion. On stopping the magnet the deflection becomes zero.

(ii) Using magnet of greater pole-strength the deflection is increased.

(iii) By increasing the number of turns on the coil or by placing a soft iron core in between, the deflection in the galvanometer is increased.

(iv) The deflection further increase by increasing the speed with which the magnet is moved towards or away from the coil i.e., increase in relative motion results in a larger deflection.

(v) The direction of deflection in the galvanometer depends on the magnetic pole which is moving towards or away from the coil.

(vi) If the magnet is kept at rest and the coil is made to move with the same speed with which the magnet was moved earlier, same amount of deflection is observed. Thus the induced e.m.f. and current developed depend on the relative motion between the magnet and the coil.

(vii) The induced e.m.f. developed does not depend on the resistance of the coil while the induced current depends on it.

In another experiment the apparatus used was as shown in Fig. (2.21) Two coils are placed side by side. One coil is connected to a cell E, resistance R and key K while the other one is connected to a sensitive galvanometer. When the key K is pressed a current flows through the first coil and at the same time a momentary deflection is observed in the galvanometer connected to the second coil. On releasing the key K the current in the first circuit is stopped but again a momentary deflection is observed in the galvanometer. The deflection on switching off is opposite to the deflection on switching on. With a steady flow of current in the first coil no defection or current is observed in the second circuit. In this experiment there is no relative motion between the two circuits but the deflection is observed on closing and opening the key. By closing or opening the key the current in the first circuit grows or decays and with that the magnetic field produced by the coil also changes.

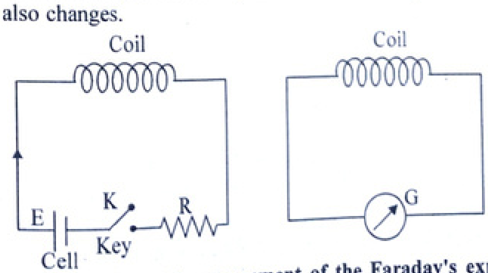

Fig. (2.21) : Experimental arrangement of the Faraday's experiment

From these experiments one can conclude that the source for the generation of e.m.f. in the coil connected to the galvanometer is the magnetic flux linked with the coil and whenever this flux changes an induced e.m.f. is developed.

12. Faraday's Laws of Electromagnetic Induction

The results obtained from the experiments on electromagnetic induction were explained by Faraday on the basis of magnetic flux linked with the circuit. For the cause of generation of induced e.m.f. and the magnitude of e.m.f. developed, Faraday gave two laws called Faraday's laws of electromagnetic induction.

(i) **Faraday's first law : Whenever the magnetic flux linked with a circuit changes with time an induced e.m.f. is developed in the circuit. The induced e.m.f. in the circuit exists so long as the change in magnetic flux continues.** This law gives the cause of the generation of induced e.m.f. If the circuit is closed then due to the induced e.m.f. an induced current is produced in the circuit.

(ii) **Faraday's second law : The magnitude of induced e.m.f. in a circuit is equal to the rate at which the magnetic flux linked with that circuit changes.** This law thus gives the magnitude of e.m.f. developed.

If in time Δt the change in magnetic flux linked with the circuit is $\Delta\phi$ then the magnitude of induced e.m.f.

$$|\epsilon| = \frac{\Delta\phi}{\Delta t}$$

and if $\Delta t \rightarrow 0$ then

$$|\epsilon| = \lim_{\Delta t \rightarrow 0} \frac{\Delta\phi}{\Delta t} = \frac{d\phi}{dt}$$

If $\frac{\Delta\phi}{\Delta t}$ i.e., rate of change of flux is in weber/second then the induced e.m.f. ϵ will be in volts.

Explanation of experimental facts by Faraday's laws : Every magnet produces a magnetic field and the lines of force outside it start from the north-pole and end up at south-pole. When a coil which is connected to a galvanometer is placed near the magnet a definite number of lines of force pass through it, as is shown in Fig. (2.22) i.e., a definite amount of magnetic flux is linked with the coil. When the magnet or the coil is moved and they are brought near each other the number of lines of force i.e., the magnetic flux passing through the coil increases. According to Faraday's laws an e.m.f. is induced in the coil due to change of magnetic flux linked with it. When the magnet or coil is moved in such a way that they go away from each other then again due to decrease in magnetic flux an

e.m.f. is developed inthe coil but now the direction is opposite to the state when they were brought nearer. Thus, due to relative motion between the magnet and the coil magnetic flux linked with the coil changes and an induced emf is developed in the coil.

The rate of change of flux will depend on the speed of relative motion between the coil and the magnet, the higher is the speed the greater will be the rate of change of flux and larger will be emf induced and vice-versa. If there is no relative motion i.e., both are either at rest or both are moving with same velocity in the same direction then the magnetic flux linked with the coil will remain constant and induced emf will be zero.

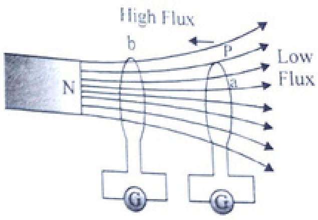

Fig. (2.22) : Experimental facts by Faraday's law

In the second experiment when the key of the first circuit was closed or opened an emf was induced in the neighbouring circuit and this can also be explained with the help of Faraday's laws. On closing the key the current in the circuit increases from zero to a maximum value and this changes the magnetic field produced by the first coil. The neighbouring coil of the second circuit is placed in the magnetic field produced by the first coil and when the current increases the magnetic flux linked with second coil increases resulting in the production of induced emf. On opening the key the current in the first circuit decreases from the maximum value to zero. Again due to this change of current the magnetic flux linked with the second coil changes and an induced emf is developed. Further at break the deflection in galvanometer connected to second coil is more than that at make, because the decay of current is faster than the growth of current.

13. Lenz's law

The production of induced emf and its magnitude are explained on the basis of Faraday's law but the knowledge of direction of induced emf and induced current is obtained with the help of another law called **Lenz's law. According to the Lenz's law the direction of induced current in a circuit is always such that it opposes the very cause which has produced it.**

When the north-pole of a magnet is brought near a coil connected to a galvanometer the direction of induced current is such that the face of the coil in-front of the north-pole of the magnet behaves like a north-pole i.e., looking into the coil from the side of the magnet the current in the coil is anti-clockwise from B to A, as shown in Fig. (2.23a). The north-pole so developed opposes the north-pole of the magnet moving towards it.

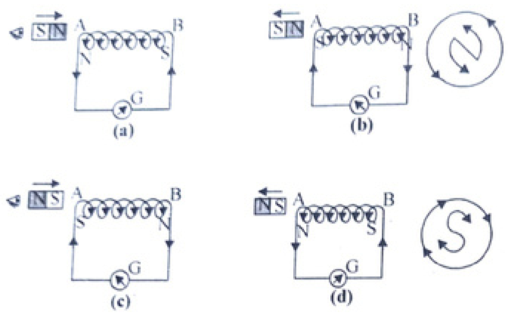

Fig. (2.23) : Lenz's Law

In the same way when the north-pole of the magnet is taken away from the coil the induced current in the coil is clockwise (A to B) so that the end A behaves like a south pole, again opposing the movement of north-pole away from it, Fig. (2.23b)

By moving the south-pole of the magnet towards the coil and away from it again the induced current developed is always such that in effect it opposes the motion of the magnet. When the south-pole is brought nearer, the end A of the coil behaves as south-pole (current clockwise from A to B) and when the south-pole is moved away the end A acts as north-pole (current anti-clockwise, from B to A) to oppose the motion of the magnet.

In the above discussion on should remember that the development of magnetic poles due to induced current (end A behaving as north-pole or south-pole) does not oppose the external magnetic field but it opposes the variation of magnetic flux.

Further, it should be noted that Lenz's law gives the direction of induced current i.e., is applicable to closed circuits only. If the circuit is open and we have to find the direction of induced emf then assuming the circuit to be closed the direction of induced current is determined which in turn gives the direction of induced emf.

It is clear that in every case for the relative motion between the magnet and coil work has to be done against the opposing force developed due to induction. This mechanical work appears as electrical energy and the total energy of the system remains conserved. The mechanical work done by the external source is equal to the energy consumed in Joule-heating and thus in electromagnetic induction the law of conservation of energy is obeyed.

Faraday's law provide the cause and magnitude of induced emf developed while Lenz's law gives its direction. Combining these laws the induced emf developed by electromagnetic induction is given by

$$\in = \frac{d\phi}{dt}$$

where the negative sign shows that the induced emf opposes the cause (change of flux) which produces it.

14. Intensity of Magnetisation (I)

The magnetic moment produced in unit volume of a substance is called intensity of magnetisation. If the magnetic moment produced in a substance of volume V is M, then

$$I = \frac{M}{V}$$

Its unit is ampere/m (A/m).

If the shape of the substance is rectangular whose cross-sectional area is A, length l and the pole strength produced in it is m, then

$$M = m \times l$$

$$\therefore \qquad I = \frac{M}{V} = \frac{m \times l}{A \times l} = \frac{m}{A}$$

Thus **the pole strength produced in a unit cross-sectional area of a bar magnet is called intensity of magnetisation.**

Intensity of magnetisation is a vector quantity. It is either along or opposite to the direction of the magnetising field (H).

In diamagnetic substances the direction of I is opposite to that of H and in paramagnetic and ferromagnetic substances the direction of I is along the direction of H. Intensity of magnetisation in a substance depends on the nature of the substance and also on temperature.

15. Magnetising Field (H)

While studying the effect of magnetic field on a substance the magnetic field, also called magnetic induction, is represented by a vector $\vec{B}$. Magnetic field $\vec{B}$ depends on the external currents as well as the magnetisation of the medium. In absence of medium, i.e. vacuum if the magnetic induction is B_0, then $\frac{B_0}{\mu_0}$ defines a vector, called magnetising field $(\vec{H})$. Magnetizing field depends only on the external currents and the geometry of the current carrying conductor.

$$\therefore \quad \text{Magnetising field } \vec{H} = \frac{\vec{B}_0}{\mu_0} \text{ A/m or A-turns/m.}$$

CGS unit of H is oersted. 1 oersted $= \frac{1}{4\pi} \times 10^3$ A/m.

For example, the magnetising field at the centre of a current carrying coil of radius a and turns n is

$$H = \frac{I}{\mu_0}\frac{\mu_0 ni}{2n} = \frac{ni}{2a}$$

and for a current carrying solenoid of n turns per unit length,

$$H = \frac{\mu_0 ni}{\mu_0} = ni$$

In media other than air or vacuum H is related to B as

$$B = \mu_0 (H + 1)$$

16. Magnetic Susceptibility (X)

When a substance is placed in an external magnetising field H, magnetic moment is induced in it. The intensity of magnetisation I is parallel or opposite to the direction of the magnetising field. In weak magnetising fields, the intensity of magnetisation I is directly probational to the magnetising field H. As a result the ratio of I and H is a constant. This constant is called **magnetic susceptibility**, i.e.,

$$I \propto H$$

or $$I = XH$$

hence $$X = \frac{1}{H}$$

where X is the magnetic susceptibility of the substance. It is a dimensionless quantity.

If $H = 1$, then $X = 1$.

The magnetic susceptibility is numerically equal to the intensity of magnetisation induced by a unit magnetising field in that substance. Magnetic susceptibility depends upon the nature of the substance. It is negative and small

for diamagnetic substances $(X \le 0)$ positive and small for paramagnetic substances $(X \ge 0)$ and positive and very large for ferromagnetic substances $(X >> 0)$.

Magnetic susceptibility is defined for unit volume of substance, so it is also called volume magnetic susceptibility. Similarly mass magnetic susceptibility is

$$X_m = \frac{X}{d}$$

and molar magnetic susceptibility is

$$X_{molar} = \frac{X}{d}A$$

where d is the density of the substance and A is the atomic weight of the substance.

17. Magnetic Permeability

The ratio of the magnetic induction B inside a substance and the magnetising field H is called magnetic permeability of the substance.

It is represented by μ. Thus

$$\mu = \left(\frac{B}{H}\right)$$

If B_0 is the magnetic induction in vacuum then the magnetic permeability of vacuum will be

$$\mu_0 = B_0/H = 4\pi \times 10^{-7}$$

Unit of μ is weber/ampere-metre or henry/m.

If μ_r is relative magnetic permeability of the substance, then

$$\mu_r = \frac{\mu}{\mu_0}$$

or $$\mu = \mu_r \mu_0$$

The magnetic permeability is always positive.

(a) For diamagnetic substances : $0 < \mu_r < 1$

(b) For paramagnetic substances : $\mu_r > 1$

(c) For ferromagnetic substance : $\mu_r >> 1$

The magnetic permeability of some substances depends on the temperature and the intensity of magnetisation. For different substances μ may be less or more than μ_0.

Magnetic permeability μ is related to the susceptibility X as

$$\mu = \mu_0 (1 + X)$$

or $$\frac{\mu}{\mu_0} = \mu_r = (1 + X)$$

18. Qualitative Explanation of Magnetic Properties

When different materials are placed in a magnetic field, they get magnetised and magnetic property of different magnitudes is produced in them. These materials are classified in the following categories on the basis of their magnetic properties :

(i) Diamagnetic. (ii) Paramagnetic and

(iii) Ferromagnetic.

(i) Diamagnetic substances : When a diamagnetic material is placed in a non-uniform magnetic field, they move from higher intensity region to lower intensity region. i.e., they are repelled by magnetic field. Magnetic moments are induced in it in small quantity and in a direction opposite to that of magnetising field. Magnetic induction B induced in these materials is less than that in free space. Magnetic permeability of these materials is positive and less than one. Magnetic susceptibility of these materials is negative and very small. It does not depend up on the temperature and the magnetising field. Diamagnetic materials are Cu, Hg, H_2O, He etc.

Diamagnetism of these materials can be explained on the basis of electron theory. In atoms electrons are considered to be moving in circular orbits around the nucleus. When an external magnetic field is applied in a direction normal to the plane of circular orbit, the motion of electron is affected, as a result the magnetic moment of the electron is changed. From analysis it is found that the angular velocity of an electron is different for clockwise and anticlockwise rotation inthe presence of the magnetic field. The magnetic moments induced due to these rotations do not cancel each other and net magnetic moment is induced in the atom whose direction is opposite to the applied magnetic field. This property of materials is called diamagnetism. Diamagnetism is a property of all substances and is produced due to orbital motion of all electrons in atoms. Since the motion of electrons does not depend upon the temperature, so the **magnetic susceptibility of the diamagnetic substances does not depend upon the temperature.**

(ii) Paramagnetic substances : When a paramagnetic substance is placed in a nonuniform magnetic field it is attracted from low intensity region to high intensity region. Magnetic moment induced in these are in the direction of magnetic field. The magnetic induction B in these materials is greater then that in free space. Their permeability is positive and greater than one. Their magnetic susceptibility is positive and very small. It does not depend up on the magnetising field but depends upon the temperature. Their dependence on temperature is according to Curie law. Paramagnetic materials are Al, Cu, Cl_2, $NiSO_4$ etc.

Inner orbits of atoms of these materials are not completely filled. They have net magnetic moment. At moderate temperatures the magnetic moments of different atoms of materials are randomly oriented, as a result the net magnetic moment of the substance is zero. When such substances are placed in a magnetic field, the atomic magnetic moments get aligned inthe direction of magnetic field and the substance is magnetised. This magnetic property is called paramagnetism. When the intensity of magnetising field is increased, the intensity of magnetisation of the substance increases but due to increase in temperature the thermal motion of the atoms tends to disturb the alignment of the atomic magnets and as a result the intensity of magnetisation decreases. Thus it is found that the magnetic susceptibility of these materials depends upon the temperature as well as the magnetising field. According to Curie's law the magnetic susceptibility X of a paramagnetic substance depends on temperature T as

$$X \propto \frac{1}{T} \text{ or } X = \frac{C}{T}$$

C is a constant called Curie's constant.

(iii) Ferromagnetic substances : When ferromagnetic materials are placed in a non-uniform magnetic field, they experience strong attraction from low intensity region to high intensity region. The magnetic moments induced in them and the intensity of magnetisation are very high and in the direction of magnetising field. The magnetic induction B in these materials is very high in comparison to that in vacuum. The magnetic permeability is positive and very large. The magnetic susceptibility is also positive and very large. Their magnetic susceptibility depends upon the temperature and the magnetising field both. Its dependance on temperature is according to Curie-Weiss law. Ferromagnetic materials are Fe, Ni, Co, Fe_2O_3 etc. Curie-Weiss law is

$$X = \frac{C}{T - T_C} \text{ where } T_C \text{ is called Curie-temperature.}$$

Curie temperature is that temperature at or below which a material behaves like a ferromagnetic material and above which it behaves like a paramagnetic material. For example Curie temperature of iron is 770°C and that of nickel is 365° C.

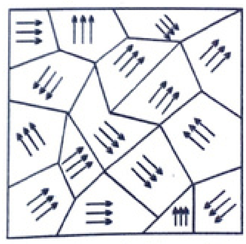

Fig. 2.24 : Domain formation of ferromagnetic materials

Ferromagnetism is explained on the basis of domain theory. Inner shells of atoms of ferromagnetic materials are partially filled and electrons of these shells are responsible for the ferromagnetism. The electrons of inner shells force the free electron to remain in the position of opposite spin. This free electron tends to bring the electron of other atom in the position of parallel spin of first atoms. In this way first atom successfully brings the other atom in the state of parallel spin with the help of free electron. Thus

small regions of parallel spin are formed in the ferromagnetic materials due to this process. These regions are called domains. In each domain all atoms are in the state of parallel spin, as a result there is a net magnetic moment in each domain. However the spin directions of other domains are different and randomly oriented, as a result net magnetic moment of the substance remains zero. In the adjoining Fig. (2.24) domain formation of ferromagnetic materials is shown.

When a ferromagnetic material is placed in a magnetic field the domains in which the direction of magnetic moments are not in the direction of magnetising field, tend to align themselves parallel to the applied magnetic field and the size of favourable domain increases. As a result intensity of magnetisation increases. If a strong magnetic field is applied, then the directions of all domains are aligned in the same direction and the large domain is formed. Hence the intensity of magnetisation becomes very high and magnetic saturation is reached in the substance.

19. Comparative Study of Magnetic Properties of Different Materials

S.No.	Property/Effect	Diamagnetic	Paramagnetic	Ferromagnetic
1.	Effect of non-uniform field	Repelled from high intensity region to low intensity region.	Attracted from low intensity region to high intensity region	Strongly attracted from low intensity region to high intensity region
2.	Magnetic moment (M)	M is nearly zero and in a direction opposite to H.	M is very small and in the direction of H.	M is very large and in the direction of H.
3.	Intensity of magnetisation (I)	I is very small and in opposite direction to H.	I is small and in the direction of H.	I is very large and in the direction of H.
4.	Magnetic induction B and number of lines of force passing through	$B < B_0$, the number of lines of force is small.	$B > B_0$, the number of lines of force slightly increases.	$B >> B_0$, the number of lines of force greatly increases.
5.	Magnetic susceptibility (X)	X is negative and very small.	X is positive and very small.	X is positive and very large.

6.	Relative permeability (μ_r)	μ_r is positive and less than one, ($\mu_r < 1$)	μ_r is positive and greater than one, ($\mu_r > 1$)	μ_r is positive and very large, $\mu_r >> 1$
7.	Dependence of X on H	No dependence	No dependence	Dependence
8.	Dependence of X on T	No dependence on T	X is inversely proportional to T (Curie's law) $X \alpha \frac{1}{T}$ or $X = \frac{C}{T}$	As temperature increases, X decreases according to Curie-Weiss Law, $X = \frac{C}{T - T_C}$
9.	Change of on type of substance to another type of substance	Do not change into another type of substance.	Paramagnetic remains paramagnetic.	On heating above T_C they are converted into paramagnetic.
10.	Explanation of magnetic property and reason of magnetism	Based on electron theory it is due to orbital motion of electrons.	Based on electron theory. It is due to orbital and spinning motion of electrons.	Based on domain theory. It is due to domain formation.
11.	Powder or liquid in watch glass placed between poles of a magnet.	Concentrates in the middle	Concentrates at the edges and is depressed in the middle	High concentration at the edges and strongly depressed in the middle.
12.	Suspending a rod between the magnetic poles	Rod aligns perpendicular to the field	Rod aligns parallel to the field	Rod aligns parallel to the field even in weak field.

20. Self Induction

Consider a conducting coil connected to a battery and a key, Fig. (2.25 a). When on closing the key a current is passed through the coil, it produces a magnetic field. As a result magnetic flux passes through the coil or magnetic flux is linked with the coil. On closing the key the current increases with time and along with it the magnetic flux linked with the coil also increases from zero to a maximum value which produces an induced emf in the coil. On breaking the circuit the current decays from the maximum value to zero and the magnetic flux also decreases from the maximum value to zero. Again an emf is induced in the coil.

According to Lenz's law the direction of induced emf in the coil is such that it opposes the change of magnetic flux linked with the coil. The directions of induced current at make and break are shown in Fig. (2.25b) and (2.25c). In general **due to a change in current passing through a coil the magnetic field produced by it and the magnetic flux linked with it change with time, producing an induced emf in the coil. This phenomenon is called self induction.**

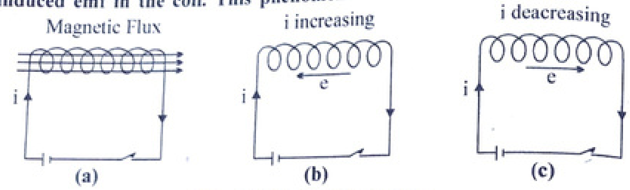

Fig. (2.25) : Self Induction

The phenomenon of self induction occurs not only in a coil but in every current carrying circuit or element. However, it is more significant and important in coils carrying current varying with time.

Coefficient of self induction or self inductance : When a current is passed through a coil the magnetic field produced by it and the magnetic flux linked with it are directly proportional to the current. If the current in the coil is i and the magnetic flux linked with it is ϕ, Then

$$\phi \propto i$$

or

$$\phi = Li \qquad ....(1)$$

where L is a constant of the coil and it is called coefficient of **self induction of self inductance.**

A change in current i produced a change in the magnetic flux ϕ linked with it. Hence according to the laws of induction an induced emf E is developed in the coil, which is given by

$$E = -\frac{d\phi}{dt} = -L\frac{di}{dt} \qquad ....(2)$$

Relation (1) and (2) can be used to define self inductance L.

From (1), $$L = \frac{\phi}{i}$$

and if i = 1 than $$L = \phi \qquad ....(3)$$

i.e., self inductance L of a coil is numerically equal to the magnetic flux linked with the coil when a unit current passes through the coil.

From (2), $$L = \frac{E}{\left(-\frac{di}{dt}\right)} \qquad ....(4)$$

If $$\left(-\frac{di}{dt}\right) = 1 \text{ than } L = E$$

i.e., the coefficient of self induction or self inductance L of a coil is numerically equal to the induced emf developed in it when the rate of decay of current in the coil is unity.

The M.K.S. unit of self inductance is henry or volt-sec/ampere i.e., weber per ampere.

$$1 \text{ henry (H)} = \frac{1 \text{volt}}{1 \text{ ampere / second}} = \frac{1 \text{ weber}}{1 \text{ ampere}}$$

If a change in current in a coil at the rate of one ampere per second produces on induced emf of one volt its coefficient of self induction or self inductance is one henry.

The unit henry is a large one so its sub-multiples are often used.

1 milli-henry (mH) = 10^{-3}H

1 micro-henry (μH) = 10^{-6} H.

The dimensions of coefficient of self induction are $M^1L^2T^{-2}A^{-2}$.

Self-inductance of a coil : The self inductance of a coil depends on its area of cross-section, number of turns and medium inside it (code).

Suppose a coil has N turns and its radius is R. If a current i flow through it the magnetic field produced at its centre is

$$B = \frac{\mu N i}{2R}$$

where μ is the permeability of the medium. If R is small, this field can be assumed uniform and the flux passing through the coil is

$$\phi_0 = BA = \frac{\mu N i}{2R}(\pi R^2)$$

If due to any reason the flux changes with time then an induced emf $-\frac{d}{dt}(BA)$ is developed in each turn of the coil in the same direction. As a result the total emf developed in the coil becomes $-N\frac{d}{dt}(BA)$. Thus a coil with N tuns behaves as if the flux passing through it is $N\phi_0 = NBA$. $N\phi_0$ is called the **flux linkage** of the coil

$\therefore$ The flux linked with the coil

$$\phi = N\phi_0 = \frac{\mu N^2 i \pi R^2}{2R}$$

Using the definition of self inductance $\left(L = \frac{\phi}{i}\right)$ the self inductance of the coil is

$$L = \frac{\pi\mu N^2 R}{2} \text{ henry.}$$

Thus L will depend on number of turns N, radius R and the permeability of the core μ.

A coil having a large self inductance L has a higher induced emf.

Self-inductance of a solenoid : Consider a solenoid of length l and area of cross-section A. The total number of turns in the solenoid is N. If a current i is passed through it, a magnetic induction B is produced along the axis of the solenoid

$$B = \mu ni = \mu \frac{N}{l} i$$

where μ is permeability of the material of core. For air $\mu = \mu_0 = 4\pi \times 10^7$ H/m.

Magnetic flux passing through the solenoid

$$\phi_0 = BA = \frac{\mu NiA}{l}$$

$\therefore$ Flux linked with the N turns of the solenoid

$$\phi = N\phi_0 = \frac{\mu N^2 iA}{l}$$

By definition of self inductance $\phi = Li$.

$$\therefore \quad Li = \frac{\mu N^2 iA}{l}$$

so that

$$L = \frac{\mu N^2 A}{l}$$

If radius of solenoid is R then $A = \pi R^2$.

$$\therefore \quad L = \frac{\pi\mu N^2 R^2}{l} \text{ henry}$$

For example for an air cored solenoid of length 40 cm, diameter 3 cm and total number of turns 500, the self inductance will be

$$L = \frac{\pi \times 4\pi \times 10^{-7} \times (500)^2 \times (1.5 \times 10^{-2})^2}{0.4}$$

$$= 550 \times 10^{-6} \text{ H} = 550\ \mu\text{H}.$$

21. Mutual Induction

Consider two coils A and B placed close to each other. The coil A is connected to a battery and a key in series. A galvanometer is connected between the ends of coil B. When the key is closed a current passes through the coil A and a magnetic field is created around this coil. The coil B is situated in this magnetic field so that magnetic flux is linked with this coil. On closing the key current in the coil. A increases from zero to a maximum value and accordingly the magnetic flux linked with the coil B increases from zero to a maximum value. Due to this change in flux an emf is induced in the coil B. In the same way when the key K is opened the current in the coil A decreases rapidly to zero so that the magnetic flux linked with coil B also decreases from the maximum value to zero. Again due to change in flux linked with coil B an emf is induced in it. In both the cases the direction of induced emf or induced current in B is such that it opposes the change in flux linked with B or the change in current in A. The direction of induced current in B at the time of break is opposite to the direction at the time of make.

When the change in current in a coil or circuit causes a change in magnetic flux linked with another neighbouring coil or circuit so that an emf is induced in the other circuit, the phenomenon is called mutual induction.

The coil in which the current is changed is called primary coil and the neighbouring coil in which an emf is induced is called secondary coil. In Fig. (2.26) coil A is primary and coil B is secondary.

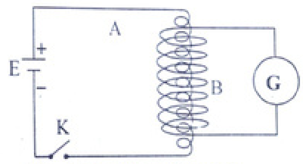

Fig. 2.26 : Experimental arrangement of mutual interaction

To have a more effective phenomenon of mutual induction the flux linkage between the coils must be as large as possible. For this purpose the two coils insulated from each other are wound over the same core of some magnetic material having a high permeability or a closed magnetic path is provided in such a way that maximum number of magnetic lines of force due to the primary coil pass through the secondary as is shown in Fig. (2.27).

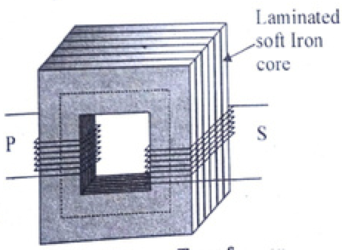

Fig. (2.27) : Transformer

The construction and working of a transformer are based on the above mentioned principles related to mutual induction.

Coefficient of mutual induction : Suppose the current in coil A at any instant is i_A and the magnetic flux linked with coil B due to magnetic field created by A is ϕ_B, then

$$\phi_B \propto i_A$$

or $$\phi_B = Mi_A \quad ....(1)$$

Here M is a constant which measures the magnetic coupling between the two coils and is called **coefficient of mutual induction.**

If $i_A = 1$, then $\phi_B = M$

i.e., the coefficient of mutual induction for two coils magnetically coupled to each other is numerically equal to the magnetic flux linked with the secondary when a unit current flows through the primary.

According to the laws of induction the emf induced in B is

$$E_B = -\frac{d\phi_B}{dt} = -M\frac{di_A}{dt}$$

so that $$M = \frac{E_B}{-(di_A/dt)}$$

$$= \frac{\text{EMF induced in secondary}}{\text{Rate of decay of current in primary}}$$

If $-\dfrac{di_A}{dt} = 1$, than $M = E_B$

i.e., the coefficient of mutual induction between two coils is numerically equal to the induced emf in the secondary due to a unit rate of change of current in the primary.

M.K.S. unit of mutual induction between two coils depends on the number of turns and the area of cross-section of the two coils, the permeability of the medium which provides the magnetic path for linkage and coupling between the coils. A larger value of M results in a Larger emf induced in the secondary.

When two coils, magnetically coupled to each other are given any one of these can be taken as primary and the other as secondary. Thus the coefficient of mutual induction

$$M_{AB} = M_{BA} = M.$$

For the estimation of M consider two coaxial solenoids each of length l having n_1 and n_2 number of turns per unit length, respectively. If the permeability of the core is μ, the magnetic induction produced by a current i_1 in the primary, is

$$B = \mu n_1 i_1 = \mu\frac{N_1}{l}i_1$$

where N_1 is the total number of turns in the primary. If the area of cross-section is A the flux passing through it is

$$\phi_0 = BA = \left(\mu\frac{N_1 i_1}{l}\right)A$$

and the flux linked with the primary coil will be

$$\phi = N_1\phi_0 = \frac{\mu N_1^2 i_1 A}{l}$$

$$= L_1 i_1 \text{ (by definition of self inductance)}$$

∴ Self inductance of primary coil

$$L_1 = \frac{\mu N_1^2 A}{l}$$

Similarly the self inductance of secondary coil

$$L_2 = \frac{\mu N_2^2 A}{l}$$

Due to mutual induction the magnetic flux linked with the second coil when a current i_1 passes through the first coil, will be

$$\phi_2 = N_2(BA)$$

$$= N_2\frac{\mu N_1 i_1 A}{l} = \frac{\mu(N_1N_2)i_1A}{l}$$

$$= Mi_1 \text{ (by definition of mutual inductance)}$$

∴ $$M = \frac{\mu N_1 N_2 A}{l} = \sqrt{L_1L_2}$$

Usually the magnetic field produced by primary is not fully linked with secondary. In such a case $M = K\sqrt{L_1L_2}$, where K is called coupling coefficient. In ideal case for perfect coupling K = 1.

22. Energy Stored in a Inductance

At a certain time if current i flows through a coil of self inductance L the emf induced in the coil due to variation of current is

$$E = -L\frac{di}{dt}$$

The work done by the induced emf against the current flow per second will be

$$-iE = Li\frac{di}{dt}$$

If current in the coil increases from 0 to i_{max}, then the total work done

$$W = \int_0^{i_{max}} Li\frac{di}{dt}\,dt = \int_0^{i_{max}} Li\,di = \frac{1}{2}Li^2_{max}$$

Total work done by the induced emf against the current is stored in the inductor in the form of magnetic energy.

$\therefore$ Energy stored in the inductor $= \frac{1}{2}Li^2_{max}$

23. Growth and Decay of Current in L-R Circuit

Consider an L-R circuit connected to a cell through a key. When the key K is pressed or closed, the current in the circuit increases but it does not attain the steady state value immediately because an opposing emf is induced in the coil which slows down the growth of current.

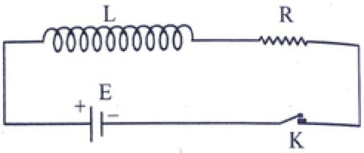

Fig. (2.28) L-R Circuit

The steady state value of current is attained after some time depending on the values of L and R.

When key is opened, the current in the circuit does not decrease to zero at once but decreases to zero after some time, because emf is induced in the coil which opposes the decay of current.

Growth of current : Let I be the current in the circuit at time t. The net emf in the circuit will be the sum of emf of cell E and emf induced in the inductance coil $\left(-L\frac{dI}{dt}\right)$. Therefore by Kirchhoff's law

$$E - L\frac{dI}{dt} = IR$$

or
$$L\frac{dI}{dt} = E - IR = -R\left(1 - \frac{E}{R}\right)$$

In the steady state when current reaches the maximum value, $\frac{dI}{dt} = 0, I = \frac{E}{R} = I_0$.

$\therefore$
$$L\frac{dI}{dt} = -R(I - I_0)$$

or
$$\frac{dI}{(I - I_0)} = -\frac{R}{L}dt$$

Integrating the above equation,

$$\log_e(I - I_0) = \frac{R}{L}t + \text{constant } K$$

Now at t = 0, I = 0, $\quad K = \log_e(-I_0)$

$\therefore$
$$\log_e(I - I_0) = -\frac{R}{L}t + \log_e(-I_0)$$

or
$$\log_e(I - I_0) - \log_e(-I_0) = -\frac{R}{L}t$$

or
$$\log_e\left(\frac{I - I_0}{-I_0}\right) = -\frac{R}{L}t$$

or
$$\frac{I - I_0}{-I_0} = e^{-Rt/L}$$

$\therefore$
$$I = I_0 - I_0 e^{-Rt/L}$$
$$= I_0(1 - e^{-Rt/L})$$

Thus the growth of current with time depends on the value of time t compared to the value of constant L/R, called time constant of the circuit. It is represented by τ.

$\therefore$
$$I = I_0(1 - e^{-t/\tau})$$

When $t = \tau = \frac{L}{R}$, $\quad I = I_0(1 - e^{-1}) = 0.632\ I_0$.

The current at t = 0 is zero and it reaches the maximum value I_0 asymptotically at $t = \infty$.

Decay of current : When the key is opened the cell emf is disconnected i.e. E = 0. If now at time t after breaking the circuit the current is I then

$$-L\frac{dI}{dt} = IR$$

or
$$\frac{dI}{I} = -\frac{R}{L}dt$$

Integrating, $\log_e I = -\frac{R}{L}t + \text{constant } K'$

Now at t = 0, the current was maximum i.e., $I = I_0$

$\therefore \quad K' = \log_e I_0$

Hence $\log_e I = -\frac{R}{L}t + \log_e I_0$

or $\log_e I - \log_e I_0 = -\frac{R}{L}t$

or $\log_e \frac{I}{I_0} = -\frac{R}{L}t$

$\therefore \quad \frac{I}{I_0} = e^{-Rt/L}$

or $I = I_0 e^{-Rt/L}$

$I = I_0 e^{-t/L}$

Thus at break at $t = 0, I = I_0$

at $t = \tau, I = I_0e^{-1} = 0.368 I_0$

and at $t = \infty, I = 0$

The growth and decay of current in a L-R circuit are shown in the following Fig. (2.29).

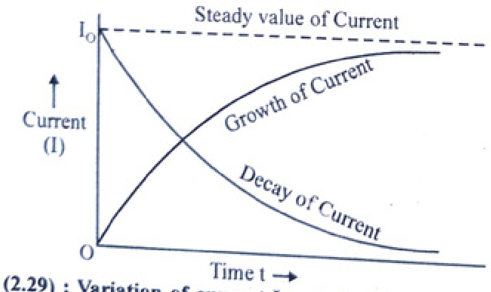

Fig. (2.29) : Variation of current I and time t in a L-R circuit

Numericals

■ **Example 1. Two identical magnetic poles of equal strength exert a force of 0.004 newton on each other when they are 8 cm apart. Find the pole strength.**

Solution : By Coulomb's law

$$F = \frac{\mu_0}{4\pi}\frac{m_1m_2}{r^2}$$

Given $m_1 = m_2 = m, r = 8 \text{ cm} = 8 \times 10^{-2}$ m.

$$F = 0.004 = 4 \times 10^{-3} \text{ N and } \frac{\mu_0}{4\pi} = 10^{-7}$$

$$\therefore \quad 4 \times 10^{-3} = 10^{-7}\frac{m^2}{(8\times10^{-2})^2}$$

or $$m^2 = \frac{4\times10^{-3}\times64\times10^{-4}}{10^{-7}} = 256$$

$\therefore \quad m = 16$ amp-m.

■ **Example 2. When a N-pole of 50 amp-m is placed 5 cm away from another pole of unknown strength it experiences a force of 2 newtons. Find the strength of unknown pole and magnetic induction at the position of given pole.**

Solution : Using Coulomb's law

$$F = \frac{\mu_0}{4\pi}\frac{m_1m_2}{r^2}$$

($m_1 = 50$ amp-m, $m_2 = m, r = 5 \text{ cm} = 5 \times 10^{-2}$ m)

$$2 = 10^{-7}\frac{50\times m}{(5\times10^{-2})^2}$$

$$m = \frac{2\times25\times10^{-4}}{10^{-7}\times50} = 1000 \text{ amp-m}$$

Magnetic induction = force per unit N-pole

$$= \frac{2}{50} = 4 \times 10^{-2} \text{ Weber/m}^2$$

■ **Example 3. The pole strength in a bar magnet is 16 amp-m and the distance between the two poles is 10 cm. Find the magnetic induction at a point on its axis at a distance of 15 cm from the N-pole.**

Solution : For a point on the axis of magnet the magnetic induction is given by

$$B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$$

Given $m = 16$ amp-m, $2l = 10$ cm $= 0.1$ m

$\therefore$ $r = 15 + 5 = 20$ cm $= 0.2$ m and $\frac{\mu_0}{4\pi} = 10^{-7}$

$M = 2ml = 16 \times 0.1 = 1.6$ amp-m^2

$$\therefore \quad B = 10^{-7}\frac{2\times1.6\times0.2}{[(0.2)^2 - (0.05)^2]^2}$$

$$= \frac{10^{-7}\times2\times1.6\times0.2}{(0.04 - 0.0025)^2}$$

$$= \frac{10^{-7}\times2\times1.6\times0.2}{14.06\times10^{-4}}$$

$= 4.45 \times 10^{-5}$ tesla

■ **Example 4. The pole strength of a short bar magnet is 15 amp-m and its effective length is 8 cm. Calculate the magnetic induction at a point on the equatorial line at a distance of 20cm from the centre of the magnet.**

Solution : For a short bar magnet $r >> l$, the magnetic induction at a point on the equatorial line is given by

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

Given $m = 15$ amp-m, $2l = 8\text{cm} = 8 \times 10^{-2}$ m

$r = 20$ cm $= 0.2$ m

$\therefore$ $M = 2ml = 15 \times 8 \times 10^{-2} = 1.2$ amp-m^2

$$\therefore \quad B = 10^{-7}\frac{1.2}{(0.2)^3} = \frac{10^{-7}\times1.2}{8\times10^{-3}} = 1.5 \times 10^{-5} \text{ tesla}$$

■ **Example 5. The radius of a coil is 10 cm and the number of turns in it is 50. If current of 0.7 A flows through it, then determine the magnetic field at the centre.**

Solution : The magnetic field at the centre of coil

$$B = \frac{\mu_0 ni}{2a}$$

$\mu_0 = 12.57 \times 10^{-7}$ Wb/A-m, $i = 0.7$A

$n = 50$, $a = 10$ cm $= 0.1$m

$$\therefore \quad B = \frac{12.57\times10^{-7}\times50\times0.7}{2\times0.1}$$

$= 2.2 \times 10^{-4}$ Wb/m^2

■ **Example 6. The radius of a coil is 0.2m and the number of turns in it is 50. How much current be passed through the coil so that the magnetic field at the centre is 5 gauss ? [1 gauss = 10^{-4} Wb/m^2]**

Solution : The magnetic field at the centre of a coil

$$B = \frac{\mu_0 ni}{2a}$$

$B = 5$ gauss $= 5 \times 10^{-4}$ Wb/m^2

$n = 50$, $a = 0.2$ m, $\mu_0 = 12.57 \times 10^{-7}$ H/m

$$i = \frac{2aB}{\mu_0 n}$$

$$= \frac{2\times0.2\times5\times10^{-4}}{12.57\times10^{-7}\times50}$$

$= 3.185$ A

■ **Example 7. The radius of a coil is 7cm and the number of turns in it is 300. A current of 0.1A is passed through the coil. Determine the magnetic field on the axis of the coil at a distance of 9cm from the centre.**

Solution :
$$\vec{B} = \frac{\mu_0 nia^2}{2(a^2 + x^2)^{3/2}}\hat{x} \text{ Wb/m}^2$$

$\mu_0 = 12.57 \times 10^{-7}$ H/m, $n = 300$, $i = 0.1$ A

$a = 7$ cm $= 0.07$ m and $x = 9$ cm $= 0.09$ m

$$\vec{B} = \frac{12.57\times10^{-7}\times300\times0.1\times(0.07)^2}{2(0.0049 + 0.0081)^{3/2}}\hat{x}$$

$= 6.23 \times 10^{-5}$ Wb/m² along the axis.

■ **Example 8. A current is flowing through a circular coil. Compare the magnetic field at the centre with the magnetic field on its axis at a distance equal to its radius.**

Solution : Suppose current i ampere is flowing through the coil of radius a. The magnetic field at the centre of the coil

$$B_1 = \frac{\mu_0 ni}{2a}$$

and the magnetic field on the axis at a distance equal to a from its centre is

$$B = \frac{\mu_0 nia^2}{2(a^2+x^2)^{3/2}}$$

$$= \frac{\mu_0 nia^2}{2(a^2+a^2)^{3/2}}$$

$$= \frac{\mu_0 ni}{4\sqrt{2}a}$$

$$\therefore \quad \frac{B_0}{B} = 2\sqrt{2}$$

■ **Example 9. Current is flowing in a coil of radius a. At what distance from the centre of the coil on the axis the magnetic field will be (1/8)th that at the centre ?**

Solution : Suppose the coil has n turns and current i is flowing through it. Thus the magnetic induction at the centre of the coil

$$B_0 = \frac{\mu_0 ni}{2a}$$

Since magnetic field at a distance x from the centre on the axis is (1/8)th of the magnetic field at the centre, so

$$B = \frac{B_0}{8} = \frac{\mu_0 nia^2}{2(a^2+x^2)^{3/2}}$$

$$\frac{1}{8} \times \frac{\mu_0 ni}{2a} = \frac{\mu_0 nia^2}{2(a^2+x^2)^{3/2}}$$

or $$\frac{1}{2^3} = \frac{a^3}{(a^2+x^2)^{3/2}}$$

or $$\frac{1}{2} = \frac{a}{(a^2+x^2)^{1/2}}$$

or $$(a^2+x^2) = 4a^2$$

or $$x^2 = 3a^2$$

or $$x = a\sqrt{3}$$

■ **Example 10. Two parallel straight wires are placed at a distance of 0.4m from each other and carry currents of 1.3 A and 2A respectively. Calculates force exerted by each wire on their unit lengths when (i) currents are in the same direction (ii) currents are opposite in direction.**

Solution : Force on unit length of wire

$$F = \frac{\mu_0 i_1 i_2}{2\pi d}$$

$\mu_0 = 12.57 \times 10^{-7}$ H/m, d = 0.4 m, I_1 = 1.3 A and I_2 = 2A

$$F = \frac{12.57\times10^{-7}\times1.3\times2}{2\times3.14\times0.4}$$

$$= 1.3 \times 10^{-6}\ N$$

(i) Attractive force F = 1.3×10^{-6} N when currents are in same direction.

(ii) Repulsive force F = 1.3×10^{-6} N when currents are in opposite direction.

■ **Example 11. Two parallel wires carry currents of 10A and 2A respectively. At what distance should they be placed from each other so that a repulsive force of 8×10^{-5} N/m acts between then ?**

Solution : Force on unit length of wire

$$F = \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$\therefore \quad d = \frac{\mu_0 i_1 i_2}{2\pi F}$$

$\mu_0 = 4\pi \times 10^{-7}$ H/m, F = 8×10^{-5} N/m, I_1 = 10A and I_2 = 2A

$$d = \frac{4\pi\times10^{-7}\times10\times2}{2\pi\times8\times10^{-5}}$$

$$= 5 \times 10^{-2}\ m$$

■ **Example 12. A solenoid has 500 turns and its length is 0.25 m. If a current of 3 A flows through it determine the magnitude of magnetic induction inside the solenoid.**

Solution : The magnetic induction inside a long solenoid is

$$B = \mu_0 \frac{NI}{L}$$

Given N = 500, L = 0.25 m, I = 3 A and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

$$\therefore \quad B = \frac{4\pi\times10^{-7}\times500\times3}{0.25}$$

$$= 7.54 \times 10^{-3}\ T$$

■ **Example 13. In an experiment a magnetic field of 10^{-3} T is to be produced in a solenoid. The length of solenoid is 0.5 m and the number of turns is 1000. Determine the value of current to be passed.**

Solution : Magnetic induction inside a solenoid

$$B = \frac{\mu_0 NI}{L} \text{ so that } I = \frac{BL}{\mu_0 N}$$

According to the question

$B = 10^{-3}$T, N = 1000, L = 0.5 m and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

$$\therefore \quad I = \frac{10^{-3}\times0.5}{4\pi\times10^{-7}\times1000} = \frac{5}{4\pi} = 0.4 \text{ ampere.}$$

■ **Example 14. In a 0.5 m long solenoid 1600 turns have been wound over an iron core. When a current of 2 A is passed the flux-density in the core is 0.8 weber/m^2. Determine the relative permeability of iron.**

Solution : The magnetic induction o flux density in the core of a long solenoid is

$$B = \mu\frac{NI}{L} = \mu_0\mu_r\frac{NI}{L}$$

Given B = 0.8 weber/m^2, N = 1600, L = 0.50 m, I = 2 A and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

$$\therefore \quad \text{Relative permeability } \mu_r = \frac{BL}{\mu_0 NI}$$

$$= \frac{0.8\times0.5}{4\pi\times10^{-7}\times1600\times2}$$

$$= \frac{10^3}{10.62} = 99.52.$$

■ **Example 15. The number of turns in a toroid of mean radius 0.10 m is 1100. Determine the magnetic induction in the toroid when a current of 2.5 A is passed through it.**

Solution : The magnetic induction in a toroid

$$B = \mu_0 \frac{NI}{2\pi R}$$

According to the question $\mu_0 = 4\pi \times 10^{-7}$ H/m, N = 1100, I = 2.5 A and R = 0.10 m.

$$\therefore \quad B = \frac{4\times3.14\times10^{-7}\times1100\times2.5}{2\times3.14\times0.10}$$

$$= 5.5 \times 10^{-3}\ T.$$

■ **Example 16. A rectangular surface of dimensions 0.04 m × 0.05 is placed in a uniform magnetic field of intensity 0.80 weber/m^2. Calculate the magnetic flux passing through this surface, when the surface is (i) along the magnetic field, (ii) perpendicular to the magnetic field and (iii) making an angle of 60° with the magnetic field.**

Solution : According to the question B = 0.80 weber/m^2 and the area of surface A = 0.04 × 0.05 = 2 × 10^{-3} m^2.

(i) When the surface is along the magnetic field then

$$\theta = 90°, \cos 90° = 0$$

$$\therefore \quad \phi = BA\cos\theta = 0.$$

(ii) When the surface is perpendicular to the field then

$$\theta = 0, \cos\theta = 1$$

$$\phi = BA\cos\theta = BA$$

$$= 0.80 \times 2 \times 10^{-3} = 1.6 \times 10^{-3} \text{ weber.}$$

(iii) When the surface makes an angle of 60° with the magnetic field then

$$\theta = 90° - 60° = 30°, \cos 30° = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \phi = BA\cos\theta$$

$$= 0.80 \times 2 \times 10^{-3} \times \frac{\sqrt{3}}{2}$$

■ **Example 17. The magnetic flux passing through a coil of 50 turns decays from 0.3 weber to zero in 1 second. Determine the emf induced in the coil.**

Solution : Given $N = 50$

Change in flux $\Delta\phi = 0 - 0.3 = -0.3$ weber

Time interval $\Delta t = 1$ second

$\therefore$ EMF induced $\in = -N\dfrac{\Delta\phi}{\Delta t}$

$$= \frac{50 \times 0.3}{1} = 15 \text{ volt.}$$

■ **Example 18. A coil of mean area of cross-section 500 cm^2 and having 1000 turns is placed in a uniform magnetic field of 4×10^{-3} tesla. If the coil is turned by 180° in 1/10 second calculate the average induced emf developed in the coil.**

Solution : When the normal to the plane of coil makes an angle θ with the magnetic field the flux linked with the coil is

$$\phi = NBA\cos\theta$$

The emf induced due to change in flux is

$$\in = -\frac{d\phi}{dt}$$

and average emf $\overline{\in} = -\dfrac{\phi_2 - \phi_1}{t}$

According to the question $A = 500\ cm^2 = 5 \times 10^{-2}\ m^2$,

$N = 1000,\ B = 4 \times 10^{-3}$ T.

By turning the coil through 180°, change in flux

$$\phi_2 - \phi_1 = NBA(\cos 180° - \cos 0°) = -2NBA$$

$$t = \frac{1}{10} = 0.1 \text{ second}$$

$$\therefore \quad \in = \frac{2NBA}{t}$$

$$= \frac{2 \times 1000 \times 4 \times 10^{-3} \times 5 \times 10^{-2}}{0.1}$$

$= 4$ volt.

■ **Example 19. The magnetic flux passing through a circuit of resistance 20 ohm changes with time in accordance with the following relation**

$$\phi = (6t^2 - 5t + 1) \text{ weber}$$

Determine (i) induced emf and (ii) induced current, at $t = \frac{1}{4}$ second.

Solution : The induced emf developed in a circuit due to change of flux is given by

$$\in = -\frac{d\phi}{dt} \text{ volt}$$

As $\phi = (6t^2 - 5t + 1)$ weber

$$\in = -\frac{d}{dt}(6t^2 - 5t + 1) = -12t + 5$$

At $t = \frac{1}{4}$ second (i) $\in = -12 \times \frac{1}{4} + 5 = 2$ volt

(ii) $i = \dfrac{\in}{R} = \dfrac{2}{20} = 0.1$ ampere.

■ **Example 20. The magnetic moment of a magnet of mass 75 gm is 8×10^{-7} A-m^2. If the density of the magnetic substance is 7500 kg/m^3, then find the intensity of magnetisation.**

Solution : $M = 8 \times 10^{-7}$ A-m^2

$d = 7500$ kg/m^3

$m = 75$ gm $= 75 \times 10^{-3}$ kg

$\therefore$ Volume $V = \dfrac{m}{d}$

$$= \frac{75 \times 10^{-3}}{7500}$$

$= 10^{-5} m^3$

$\therefore$ Intensity of magnetisation

$$I = \frac{M}{V}$$

$$= \frac{8 \times 10^{-7}}{10^{-5}}$$

$= 0.08$ A-m.

■ **Example 21. A magnetic moment of 5 A-m^2 is produced in a rod of magnetic material when placed in a magnetising field of 0.4×10^3 A/m. If the volume of the rod is $2.5 \times 10^{-5}\ m^3$, then find the intensity of magnetisation and the magnetic induction.**

Solution : $H = 0.4 \times 10^3$ A/m

$M = 5$ A-m^2

$V = 2.5 \times 10^{-5}$m^2

∴ Intensity of magnetisation

$$I = \frac{M}{V} = \frac{5}{2.5\times10^{-5}} = 2 \times 10^5 \text{ A/m}$$

$$B = \mu_0 (H + I) = 12.57 \times 10^{-7} (0.4 \times 10^3 + 2 \times 10^5) = 0.252 \text{ T}$$

■ **Example 22. A magnetising field of 2×10^3 A/m produces a magnetic flux of 6.28×10^{-4} Wb in an iron rod. The area of cross-section of rod is 2×10^{-5} m^2. Calculate the relative permeability of the rod and the intensity of magnetisation.**

Solution : $H = 2 \times 10^3$ A/m; $\phi = 6.28 \times 10^{-4}$ Wb

$A = 2 \times 10^{-5}$ m^2

∴ Magnetic flux density

$$B = \frac{\phi}{A} = \frac{6.28\times10^{-4}}{2\times10^{-5}} = 31.4 \text{ Wb/m}^2$$

Permeability $\mu = \frac{B}{H} = \frac{31.4}{2\times10^3}$

Relative permeability $\mu_r = \frac{\mu}{\mu_0}$

$$= \frac{31.4}{2\times10^3\times4\times3.14\times10^{-7}} = 1.25 \times 10^4$$

Intensity of magnetisation $I = \frac{B}{\mu_0} - H$

$$\therefore \quad I = \frac{31.4}{4\times3.14\times10^{-7}} - 2 \times 10^3 = 2.5 \times 10^7 - 2 \times 10^3 = 24998 \times 10^3 = 2.5 \times 10^7 \text{ A/m}$$

■ **Example 23. The self inductance of a coil is 3 mH and a current of 5 A is flowing through it. If the current reduces to zero on switching off in time 0.1 s, what will be the average emf developed in the coil ?**

Solution : The emf induced in the coil

$$\epsilon = -L\frac{di}{dt}$$

Average emf $\bar{\epsilon} = -\frac{\Delta i}{\Delta t}$

Given : $L = 3 \times 10^{-3}$ H, $\Delta I = 0 - 5 = -5$A, $\Delta t = 0.1$ s

$$\therefore \quad \bar{\epsilon} = -\frac{3\times10^{-3}\times(-5)}{0.1} = 0.15 \text{ volt.}$$

■ **Example 24. On passing a direct current of 2.5 amperes in a coil having 500 turns the flux passing through it is 1.4×10^{-4} weber. What is the inductance of the coil ?**

Solution : The magnetic flux linked with a coil

$$\phi = N\phi_0 = Li$$

or $$L = \frac{N\phi_0}{i}$$

Given $\phi_0 = 1.4 \times 10^{-4}$ weber, $i = 2.5$ A, $N = 500$

$$\therefore \quad L = \frac{500\times1.4\times10^{-4}}{2.5} = 2.8 \times 10^{-2} \text{ H} = 28 \text{ mH.}$$

■ **Example 25. A solenoid has a self inductance of 50H and a resistance of 25 ohm. If it is connected to a battery of 100 volt, calculate the time during which the current grows from zero to half of its maximum value.**

Solution : $I = I_0(1 - e^{-t/(L/R)})$

$$\frac{1}{2} = 1 - e^{-tR/L}$$

$$\therefore \quad t = \frac{L}{R}\log_e e = \frac{L}{R}(2.3 \log_{10}2) = \frac{50\times2.3\times0.3}{25} = 1.38 \approx 1.4\text{s}$$

Questions

Short Answer Type Question

1. Define Magnetic induction in terms of magnetic flux.
2. If a magnetic pole of strength m is placed in a magnetic field of intensity B, what is the force acting on it ?
3. What is the value and unit of permeability of free space ?
4. A bar magnet of magnetic moment M, length 2*l* and width 2d is cut into four pieces each of length *l* and width d, what will be magnetic moment of each piece ?
5. In the end-on position if the distance of the point from the centre of a short magnet is doubled what will happen to the magnetic field intensity ?
6. The intensity of magnetic field at a point on the equatorial line of a small magnet is B. Keeping the distance same what will be intensity of magnetic field if the point lies on the axis ?
7. On what factors does the magnetic induction due to a current element depend ?
8. What is the direction of magnetic field due to a current element ?
9. If a current of 1 ampere flows in a coil of 1 turn and 1 m radius, what will be the value of magnetic induction at its centre ?
10. What is the magnetic field intensity at the middle of two current carrying parallel conductors ?
11. What are the dimensions of magnetic field ?
12. How does the magnetic field inside a long current carrying solenoid vary with distance from its axis ?
13. Keeping the total number of turns fixed how will the magnetics induction inside a toroid depend on its radius ?
14. What is magnetic flux ?
15. When the magnetic flux passing through a coil changes, is the emf or current always induced ?
16. A copper ring is suspended by a thread in a vertical plane. One end of a magnet is swiftly brought towards the ring in the horizontal direction. Will the motion of magnet affect the ring ?
17. A piece of metal and a piece of non-metal are dropped from the same height above the surface of earth. Which one will reach the surface first ?
18. The resistance wires fixed in resistance boxes are first doubled, why ?
19. Two iron bars are identical in shape and size but one is magnet and other is not. How will you distinguish them without suspending or any other instrument ?

20. What is Curie temperature ?
21. What is time constant of R-L circuit ?
22. What will be the effect on growth and decay of current in a R-L circuit, if inductance L is increased ?

Long Answer Type Question

1. Obtain an expression of the magnetic induction due to a bar magnet at a point on its axis.

   For a short bar magnet how the magnetic induction depends on the distance from the centre ?
2. Obtain an expression for the intensity of magnetic field due to a bar magnet at a point on its equatorial line.

   For a short bar magnet show that the field in end-on position is double the field in broad side-on position for the same distance.
3. State Biot-Savart's law. How is the direction of magnetic field due to a current element determined ? **[Raj., 2003, 2005, 2007, 2008, 2010, 2011]**
4. Determine the magnetic field at the centre of a current carrying coil. **[Raj., 2003, 2005]**
5. Obtain an expression for the magnetic induction at a point on the axis of a current carrying coil. How does the field very with distance from the centre, show graphically. **[Raj. 2004, 2011]**
6. On what factors does the magnetic induction due to a long current carrying wire depend ? **[Raj. 2011]**

   Determine the force per unit length between two current carrying parallel conductors. Hence define the unit ampere.
7. Give the expression for magnetic induction inside a long straight solenoid. What is a toroid ? How will the field inside a toroid depend on its mean radius for a given number of turns ? **[Raj. 2006, 2008]**
8. Define magnetic flux.

   State Faraday's laws of electromagnetic induction. How do these laws explain the observations of Faraday's experiments ? **[Raj. 2006]**
9. State Lenz's law. Explain with the help of examples how does this law enable one to determine the direction of inducted current in a coil. **[Raj. 2010]**
10. Considering the response of various materials to a magnetic field, classify these in dia, para and ferromagnetic materials.
11. State the difference in the properties of dia, para and ferromagnetic substances. How will you differentiate between para and ferromagnetic materials ? **[Raj. 2002, 2004, 2008]**

12. How do the magnetic susceptibilities of dia, para and ferromagnetic materials depend on temperature ?
What is Curie temperature ? What is its value for iron ?

13. Define self induction and coefficient of self induction. Obtain an expression for the inductance of a coil. **[Raj. 2010]**

14. Define mutual induction. How is coefficient of mutual induction related to the coefficients of self induction of two coils magnetically coupled ?

15. Obtain an expression for the energy stored in an inductor. In what form does this energy exist ?

16. Discuss the growth and decay of current in an L-R circuit. What is time constant of such a circuit ?

Numerical

1. Calculate the intensity of magnetic field due to a magnetic pole of strength 16 A-m at a distance of 10 cm from it. **[Answer :** 1.6×10^{-4} T]

2. The pole strength of a short bar magnet is 10 A-m and its effective length is 4 cm. determine the magnetic induction at a distance of 20 cm from its centre in (i) the end on position, (ii) the broad side-on position.
[Answer : (i) 10×10^{-6} T, (ii) 5×10^{-6} T]

3. The radius of a coil is 10 cm and it has 100 turns. A current of 0.5A is passed through it. Determine the magnetic induction at its centre.
[Answer : 3.14×10^{-4} T]

4. The radius of a circular coil is 6 cm and number of turns is 20. If 1.5 A current is passed through the coil, then determine the magnetic field on the axis of the coil at a distance of 8 cm from its centre.
[Answer : 6.78×10^{-3} tesla]

5. The radius of a coil is R. At what distance from the centre is the magnetic field equal to (1/8) th of the magnetic field at the centre ? **[Answer :** $R\sqrt{3}$]

6. A current of 10 amperes is flowing in a long straight wire. What will be the magnetic induction at a distance of 2 cm from the wire ? **[Answer :** 10^{-4} T]

7. The number of turns in a 60 cm long solenoid is 1000. To obtain magnetic induction of 4×10^{-2} tesla inside the solenoid, how much current must be passed through it ? **[Answer :** 19.1 A] **[Raj. 2004]**

8. The mean radius of a toroid made on a copper ring is 10 cm and the number of turns in it is 500. If a current of 0.1 ampere is passed, determine the magnetic induction produced in the toroid.
[Answer : 10^{-4} T]

9. A solenoid is 100 cm long. Its diameter is 10 cm and the number of turns is 500. If a current of 2.4 ampere is passed through it then what will be the value of B at its axis ?
[Answer : 1.5×10^{-3} T]

10. By passing a current of 10 amperes in a solenoid of length 10 cm, magnetic induction of $4\pi \times 10^{-3}$ tesla is produced on it axis. Find the number of turns in the solenoid.
[Answer : 100] **[Raj. 2003]**

11. Two long parallel wires are 5 cm apart. The force on a unit length of each wire due to the other is 4×10^{-5} N/m. If the current in one wire is 2A, calculate the current flowing through the other wire. **[Answer :** 5A]

12. Two long parallel wires 2 cm apart, have current 1 A and 2A respectively in the same direction. Calculate the resultant magnetic induction at a point in the middle of the line joining the two wires perpendicularly.
[Answer : 2×10^{-5} T]

13. If the directions of currents in the wires described in Q. (12) are opposite, what will be the resultant field ? **[Answer :** 6×10^{-5} T]

14. A rectangle loop of area 0.2 m^2 is situated in a magnetic field of intensity 3×10^{-3} tesla. What will be the magnetic flux passing through the loop when
(a) the plane of the loop is perpendicular to the magnetic field,
(b) The plane of the loop makes an angle of 30 ° with the magnetic field,
(c) the plane of the loop is parallel to the magnetic field ?
[Answer : (a) 0.6×10^{-3} weber, (b) 0.3×10^{-3} weber, (c) zero]

15. A square coil has 100 turns and its side is of 0.4 m. It is placed perpendicular to magnetic field. If the intensity of field increases from 0.1 tesla to 0.5 tesla in 0.02 s, what will be the value of induced emf ?
[Answer : 320 volts]

16. The magnetic moment of an iron bar of mass 80 gm is 20 A-m^2. If density of iron is 8 gm/cm^3, then find the intensity of magnetisation.
[Answer : 10^6 A/M]

17. An iron bar is place in a magnetic field of 20 oersted. If the total magnetic flux is 0.2 Wb/m^2, then calculate the permeability, the susceptibility and the intensity of magnetisation.
[Answer : 12.57×10^{-5} H/m, 99, 157.5×10^3 A/m]

18. Calculating the magnetic induction inside a solenoid of 50 cm long and number of twins per unit length is 100, the current flowing in solenoid is 10 amp.
[Answer : 1.256×10^{-3} Tesla]

■■■