CbH

Chapter 11

Permutations and Combinations

11.1. Introduction

In algebra, we face many such problems in which we require arrangement or selection of objects. To solve such problems easily concepts of Permutations and Combinations are used.

11.2. Factorial Notation

The continued product of first n natural numbers is known as '*factorial n*' and it is denoted as $\lfloor n$ or $n!$.

i.e. $n! = 1 \times 2 \times 3 \times \ldots\ldots \times n$

For example $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Note : (i) $0! = 1$

(ii) $1! = 1$

(iii) Factorial of negative numbers is undefined.

(iv) $n! = n \times (n-1)!$

11.3. Permutation

Permutations means arrangement. The number of arrangements possible taking some or all objects from given number of objects is known as Permutation.

If we have n number of distinct objects and we have to arrange r objects out of them then number of such permutations will be ^nP_r or $P(n, r)$.

Value of $P(n, r)$ is obtained by the following formula :

$$P(n, r) = \frac{n!}{(n-r)!}$$

or

$$P(n, r) = n(n-1)(n-2) \ldots\ldots (n-r+1)$$

Example : Number of permutaitons of 4 objects taken out of 6 objects = $\frac{6!}{(6-4)!}$

$$\frac{6!}{2!} = \frac{720}{2} = 360$$

Note : (i) Number of permutations of n different objects taking all of them will be $P(n, n) = \frac{n!}{0!} = n!$ $(\because 0! = 1)$

(ii) number of permutations taking r objets out of n provided any object can be repeated any number of times will be n^r.

Example : Number of methods of distributing 3 prizes among 4 players provided any player can take more than one prizes will be = $4^3 = 64$.

11.4. Permutations of those objects in which not all distinct

Theorem : Let there be n objects in which p are of one kind, q are of second kind, r are or third kind and remaining are different then total number of permutations will be $\frac{n!}{p!q!r!}$

11.5. Circular/Cyclic permutations

If n distinct objects are arranged around a circle then such permutations are known as circular or cyclic permutations. In such permutations one object is kept stationary and other's arrangement is changed.

Theorem : Number of cyclic permutations from n distinct objects is $(n-1)!$

Note : (i) In this theorem clock wise and anticlocwise permutations are assumed to be different.

(ii) If clockwise and anticlockwise permutations are not assumed to be different then total number of cyclic permutations will be $(n-1)!/2$.

For example : In a necklace of beads clock wise and anticlockwise permutations are not assumed different.

Illustrative Examples

Example 1 : Prove the following

(i) $^nP_n = {}^nP_{n-1}$ **(ii)** $^nP_r = (n-r+1)\ {}^nP_{r-1}$

Slove

Solution : (i) $^nP_n = n!$

and $^nP_{n-1} = \frac{n!}{(n-(n-1))!} = \frac{n!}{1!} = n!$

$\therefore$ $^nP_n = {}^nP_{n-1}$

(ii) R.H.S. $= (n-r+1) \cdot {}^nP_{r-1}$

$$= \frac{(n-r+1)n!}{\{n-(r-1)\}!}$$

$$= \frac{(n-r+1).n!}{(n-r+1)!}$$

$$= \frac{(n-r+1).n!}{(n-r+1).(n-r)!} \qquad [\because n! = n(n-1)!]$$

$$= \frac{n!}{(n-r)!}$$

$$= {}^nP_r = \text{L.H.S.}$$

Example 2 : Find the value of n
(i) 2. ${}^5P_3 = {}^nP_4$ (ii) ${}^nP_5 : {}^nP_3 = 2 : 1$

Slove

Solution : (i) 2. ${}^5P_3 = {}^nP_4$

$$\Rightarrow \quad 2.\frac{5!}{2!} = \frac{n!}{(n-4)!}$$

$$\Rightarrow \quad 5! = \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!}$$

$$\Rightarrow \quad 5.4.3.2.1 = n(n-1)(n-2)(n-3)$$

$$\Rightarrow \quad 5(5-1).(5-2).(5-3) = n(n-1)(n-2)(n-3)$$

Comparing $\quad n = 5$

(ii)
$$\frac{{}^nP_5}{{}^nP_3} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{n!}{(n-5)!} \times \frac{(n-3)!}{n!} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 2$$

$$\Rightarrow \quad (n-3)(n-4) = 2.1$$

$$\Rightarrow \quad (n-3)(n-4) = (5-3).(5-4)$$

$$\Rightarrow \quad n = 5 \qquad \text{(Comparing)}$$

Example 3 : Find the number of different words which can be formed using the letters of the words MATHEMATICS.

Solution : There are 11 letters in the word MATHEMATICS. Out of which M, A, T are repeated twice, therefore total number of words will be

$$= \frac{11!}{2!2!2!}$$

Example 4 : In how may ways can 8 persons be arranged around a table ?

Solution : Required number of permutations

$$= (8-1)!$$

$$= 7!$$

$$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 5040$$

N.S

Example 5 : How many different words can be formed with the letters of PARROT if P and A do not come together.

Solution : There are 6 letters in the word PARROT out of which R is repeated twice.

$\therefore$ Total number of permutations $= \frac{6!}{2!} = 360$

If P and A come together then they can be assumed one letter and we will have 5 letters. P and A can be written as (PA) or (AP) hence total number of methods keeping them together

$$= 2! \times \frac{5!}{2!} = 120$$

$\therefore$ Now the required number of words in which P and A do not come together

$$= 360 - 120 = 240$$

Example 6 : How many 6 digit numbers may be formed using the digits 0, 1, 2, 3, 4, 5 if repetition is not allowed ?

N.S

Solution : We have 6 digits 0, 1, 2, 3, 4, 5. Using these digits we may form 6! six digit numbers but these will also include the numbers starting with 0. Number of those numbers which start with 0 will be 5!

Hence excluding these numbers, required number of six digit numbers

check

$$= 6! - 5! = 600$$

Example 7 : In how many ways 10 different beads may be arranged in a necklace so that 4 specific beads are always together ?

N.S

Solution : If 4 specific beads are always together, we can assume them as one bead. Hence total number of beads will be

$$= (10-4) + 1 = 7$$

Numbers of ways to arrange 7 beads in a necklace

$$= \frac{(7-1)!}{2} = \frac{6!}{2}$$

But those 4 beads can also be arranged in 4! ways hence required number of permutations $= \frac{6!}{2} \times 4!$

11.6. COMBINATION

Definition : Combination is defined as number of selections taking some or all objects out of a given number of objects, not considering the order of selection. Number of selections of r objects out of n object is denoted by $C(n, r)$ or ${}^nC_r$

Value of $C(n, r)$ is given by the following formula

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

or
$$C(n, r) = \frac{n(n-1)(n-2)(n-r+1)}{1.2.3......(r-1).r}$$

Example : Number of selections of 4 objects out of 6

$$= \frac{6!}{(6-4)!4!}$$
$$= \frac{6!}{2!4!}$$
$$= \frac{6\times5\times4!}{2\times1\times4!} = 15$$

Note : (i) $${}^nC_n = \frac{n!}{(n-n)!n!} = \frac{n!}{0!n!} = 1 \qquad (\because 0! = 1)$$

(ii) $${}^nC_0 = \frac{n!}{(n-0)!0!} = \frac{n!}{0!n!} = 1$$

$\therefore$ $${}^nC_n = {}^nC_0 = 1$$

11.7. Difference between Permutations and Combinations

Order of objects is very important in permutations whether in combinations we do not consider it. For example AB and BA will be considered as a single combination but these are two different permutations.

Generally in forming numbers with given digits, forming words with given letters we have to use Permutations while to select team members, to form a committee, to make groups of people or objects, to select particular letters from given words we use Combinations.

11.8. Properties of ${}^nC_r$

1. $${}^nC_r = {}^nC_{n-r} \qquad (0 \le r \le n)$$

Proof : RHS ${}^nC_{n-r} =$

$$= \frac{n!}{\{n-(n-r)\}!(n-r)!} = \frac{n!}{r!(n-r)!}$$
$$= {}^nC_r = \text{L.H.S.}$$

2. N.S $${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

Proof : LHS $= {}^nC_r + {}^nC_{r-1}$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$
$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$
$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{(n-r+1)}\right]$$
$$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$
$$= \frac{(n+1)n!}{r(r-1)!(n-r+1)(n-r)!}$$
$$= \frac{(n+1)!}{r!(n-r+1)!}$$
$$= \frac{(n+1)!}{r!\{(n+1)-r\}!} = {}^{n+1}C_r$$
$$= \text{RHS}$$

3. $${}^nC_x = {}^nC_y \Rightarrow x = y \quad \text{or} \quad x + y = n$$

Proof :
$${}^nC_x = {}^nC_y$$
$$\Rightarrow \quad {}^nC_x = {}^nC_y = {}^nC_{n-y}$$
$$\Rightarrow \quad x = y \quad \text{or} \quad x = n - y$$
$$\Rightarrow \quad x = y \quad \text{or} \quad x + y = n$$

11.9. Restricted combinations

1. Number of combinations of r objects out of n objects when p specific objects are always included

$$= {}^{n-p}C_{r-p}$$

2. Number of combinations of r objecs out of n objects when p specific objects are never included

$$= {}^{n-p}C_r$$

11.10. Division in groups

1. **Division in 2 groups :** Number of ways to divide $(p + q)$ objects in two groups of p and q objects $= {}^{p+q}C_p = {}^{p+q}C_q = \dfrac{(p+q)!}{p!q!}$

2. **Division in 3 Groups :** Number of ways to divide $(p + q + r)$ objects in

three groups of p, q and r objects $= \dfrac{(p+q+r)!}{p!q!r!}$

Illustrative Examples

Important

Example 1 : If $^nC_{15} = {}^nC_8$ then find the value of $^nC_{21}$

Solution : $^nC_{15} = {}^nC_8 \Rightarrow n = 15 + 8 = 23$

$$\therefore \quad {}^nC_{21} = {}^{23}C_{21} = \frac{23!}{21!2!}$$

$$= \frac{23\times 22\times 21!}{21!\times 2!}$$

$$= 23 \times 11 = 253$$

easy

Example 2 : How many different words may be formed taking 3 consonants and 2 vowels out of 5 consonants and 4 vowels ?

Solution : Number of ways to choose 3 consonants out of 5 $= {}^5C_3 = 10$

Number of ways to choose 2 vowels out of 4 $= {}^4C_2 = 6$

Number of ways to arrange these 5 letters

$$= 5! = 120$$

Hence required number of words $= 10 \times 6 \times 120$

$$= 7200$$

Example 3 : How many 4 digit numbers may be formed using the digits 1, 2, 3, 4, 5, 6 if 4 and 5 are always present in each number ?

Solution : Total digits = 6

Number of ways to choose 4 digits if 4 and 5 are present in each number

$$= {}^{(6-2)}C_{(4-2)} = {}^4C_2 = 6$$

Number of ways to arrange these 4 digits

$$= 4! = 24$$

$\therefore$ Total number of required numbers

$$= 6 \times 24 = 144.$$

Example 4 : A group of 7 members is to be formed from 6 boys and 4 girls such that boys are in majority in the group. In how many ways is it possible?

Solution : Since boys are in majority in the group so the following cases are possible:

Case	Number of combinations
6 boys, 1 Girl	$^6C_6 \times {}^4C_1 = 4$
5 boys, 2 Girls	$^6C_5 \times {}^4C_2 = 36$
4 boys, 3 Girls	$^6C_4 \times {}^4C_3 = 60$

$\therefore$ Number of total groups $= 4 + 36 + 60 = 100$

Example 5 : How many signals are possible taking one or more flags out of 6 flags of different colours ?

Solution : First we have to choose one or more flags out of six and then to arrange them.

So number of required signals

$$= ({}^6C_1 \times 1!) + ({}^6C_2 \times 2!) + ({}^6C_3 \times 3!) + ({}^6C_4 \times 4!) + ({}^6C_5 \times 5!) + ({}^6C_6 \times 6!)$$

$$= 6 + 30 + 120 + 360 + 720 + 720$$

$$= 1956$$

Example 6 : How many 5digit even numbers can be formed **[R.U. 2016]**

Solution : If we suppose that repeatition of digits is not allowed then total number of even numbers of 5 digits.

$$= (6\times7\times8\times9\times5) - \underbrace{(1\times6\times7\times8\times4)}_{\text{Starting with zero}}$$

$$= 13776$$

If repeatition of digits is allowed then total number of even numbers of 5 digits

$$= 9 \times 10 \times 10 \times 10 \times 5$$

$$= 45000$$

Example 7 : How many different words can be formed by the word "COMMERCE" when all vowels are not together. **[R.U. 2016]**

Imp

Solution : Total number of words $= \dfrac{8!}{2!\ 2!\ 2!}$

$$= 5040$$

If all vowels are together then total number of words

$$= \frac{6!}{2!\ 2!} \times \frac{3!}{2!} = 540$$

If vowels are not together then total number of words $= 5040 - 540 = 4500$

Example 8 : In how many different ways can a group of 4 members be formed from 4 doctors, 5 engineers and 3 teachers when at least one person of each category is sduted. **[R.U. 2016]**

Solution : Possiblities are

1 Doctor 1 Engineer 2 Teachers

1 Doctor 2 Engineers 1 Teacher

2 Doctors 1 Engineer 1 Teacher

$\therefore$ Total number of ways $= ({}^4C_1 \times {}^5C_1 \times {}^3C_2) + ({}^4C_1 \times {}^5C_2 \times {}^3C_1) + ({}^4C_2 + {}^5C_1 \times {}^3C_1)$

$= 60 + 120 + 90 = 270$

Example 9 : If $C(n_1 5) = C(n_1 16)$ then find $C(25, n)$ **[R.U. 2016]**

Solution : $\because {}^nC_5 = {}^nC_{16} \Rightarrow n = 16 + 5 = 21$

Now ${}^{25}C_n = {}^{25}C_{21} = {}^{25}C_4 = 12650$

EXERCISES 11.1

1. If ${}^{10}P_r = 5040$ then find value of r.
2. If ${}^{2n}C_3 : {}^nC_3 = 11 : 1$ then find value of n.
3. In how many ways 5 boys and 5 girls may be arranged around a table so that no two girls sit together?
4. How many three digit numbers may be formed using the digits 1,2,3,4,5,6 if repetition of digits is not allowed?
5. In how many ways the letters of the word COMBINE may be arranged so that the vowels O, I and E are always together?
6. How many different committees may be formed taking 3 men and 3 women out of 12 men and 16 women?
7. There are n points in a plane out of which m are collinear. How many triangles may be drawn with these points ?
8. Evaluate ${}^{50}C_{11} + {}^{50}C_{12} + {}^{51}C_{13} - {}^{52}C_{13}$
9. How many different words may be formed with the letters of the words SALOON if both O's don't come together?
10. Find the number of diagonals in a n-sided polygon.
11. In how many ways can 5 boys and 3 girls sit on 8 chairs if all girls sit together. **[R.U. 2016]**

ANSWERS 11.1

(1) $r = 4$	(2) $n = 6$	(3) $5! \times 4!$
(4) 156	(5) 720	(6) 1, 23, 200
(7) ${}^nC_3 - {}^mC_3$	(8) 0	(9) 240
(10) $\dfrac{n(n-3)}{2}$		

Chapter 12 Probability

12.1 INTRODUCTION

In our daily life many such incidents take place which have more than one results. It is natural that everyone has a curiosity to know the result. The science of knowing the result of an incident on the basis of proper information and circumstances is called Probability.

The theory of probability was first propounded in the 17^{th} century in Europe. The gamblers and match fixers made attempts to know the results of their respective games before hand in order to have maximum advantage. These people put this problem before their contemporary Mathematicians like Galilee, Pascal etc. These mathematicians developed certain mathematical methods to solve these problems and consequently this branch of Mathematics came to exit. Prominent mathematicians of the 18^{th} and 19^{th} centuries Laplace, Gauss, Bernoulli, etc, developed this principle further.

In modern times theory of probability is applied in various fields where decisions pertaining to future have to be taken. For example in preparing the budget of any state or country, theory of probability is used. Insurance companies prepare death tables and make inference as to how any person of particular age group is likely to survive and defence experts frame their strategies with the help of this theory. Many important policies in the fields of society, state administration, commerce and science are determined broadly on the basis of probability. First of all, we will attempt to define certain important terms used in the study of probability

12.2 SOME DEFINITIONS

1. Random Experiment : When all the possible results of an experiment are already known and no inference of any particular result is possible, it is called a random experiment.

For example, The two results of tossing a coin either head or tail are already known. No definite result can be forecast therefore, toss of a coin is a random experiment.

2. Trial and Event : When out of many possible results (outcomes) of a random experiment, one is certain, the experiment is called a trial and possible results are called events. For example :

(i) Tossing of a coin is trial and getting head (H) or tail (T) and events.

(ii) Throwing a dice is trial and getting any number 1, 2, 3, 4, 5 and 6 is event.

(iii) Appearing in the examination of a candidate is trial and to pass or fail is event.

3. Simple Event : When in a trial, only one event takes place at a time, it is a simple event for example : Drawing a ball from a bag containing a few black and white balls, is a simple event.

4. Exhaustive events or total number of cases : All possible results of trial are called Exhaustive events or total number of cases of that trial. For example :

(i) Tossing of a coin is a trial and head or tail can occur. So in this trial exhaustive events are 2.

(ii) On throwing of a die 1, 2, 3, 4, 5 or 6 can occur so in this trial exhaustive events are 6.

5. Favourable events or cases : The number of cases favourable to a particular event in a trial is the number of outcomes which entail the happening of the particular event. For example :

(i) In throwing a die, the number of cases favourable to getting an even number is 2, 4, 6 i.e. 3.

(ii) On drawing two cards from a pack of card, the number of cases favorable to getting king is $^4C_2$ i.e. 6.

(iii) In throwing of two dice, the number of cases favorable to getting a sum of 5 is (1, 4), (4, 1), (2, 3), (3, 2) i.e. 4.

6. Independent and dependent events :

(i) Independent events : Two or more events are called independent events if the happening or not happening of any one does not depend on the happening or non happening of the other. For example, on tossing a coin and throwing a die, the outcomes getting head on coin and 4 on dice are independent events.

(ii) Dependent event : Two or more events are called dependent events if the happening of any one does depend on the happenings of the other. For example :

A card drawn from an ordinary pack of cards should be a heart card, after without replacing it in pack, again a drawn card should be a spade card, both are dependent events.

7. Mutually exclusive or disjoint events : Two or more events are said to be mutually exclusive or disjoint events if no two or more occur simultaneously in the same trial i.e. if the occurrence of any one of them prevents the occurrence of all others. For example:

(i) On tossing of a coin occurring of head or tail are mutually exclusive events.

(ii) A card is drawn from a pack of card, it being a king or a queen are mutually exclusive events.

8. Equally likely events : If in an experiment, possibility of happening of all events is same then such events are called equally likely events. For example :

(i) In tossing a coin, getting a head or tail are equally likely events.

(ii) In drawing a card from a pack of cards it will be red or black card, are equally likely events.

9. Compound events : If two or more events happen at a time then they are called compound events or joint event. For example

In two bags, there are some blue and some red balls. Selection of a bag and then drawing a ball from it is compound event because selection of one bag from two bags and then a ball is drawn from selected bag is happening at a time.

10. Sample point and sample space : Each outcome of a trial is called sample point and set of all sample points of a trial is called its sample space. It is generally denoted by S.

(i) The sample point in tossing of two coins are (H, H), (H, T), (T, H), (T, T) and $S = [(H, H), (H, T), (T, H), (T, T)]$ is sample space.

Mathematical definition of Probability : If a trial results in an equally likely, mutually exclusive and exhaustive cases and m of them are favourable to the happening of an event A, then the probability of A is defined as the ratio $\frac{m}{n}$ and is denoted by $P(A)$.

Thus
$$P(A) = \frac{\text{Favourable cases of } A}{\text{exhaustive cases of } A}$$
$$= \frac{m}{n} \quad \text{(numerical measure)}$$

If in a trial, happening of event A is sure then $m = n$ and
$$P(A) = \frac{n}{n} = 1,$$

If happening of event A is impossible then m = 0 and
$$P(A) = \frac{0}{n} = 0,$$

Therefore, for any event A, $0 \le P(A) \le 1$

i.e., probability of any event can not be less then 0 and greater than 1 and limit of probability is from 0 to 1. Probability of non happening of event A is denoted as $P(\bar{A})$.

So $$P(\bar{A}) = \frac{\text{unfavourable cases of event } A}{\text{exhaustive cases of event } A}$$

$$= \frac{n-m}{n} = 1 - \frac{m}{n}$$

$$\Rightarrow \quad P(\bar{A}) = 1 - P(A)$$

Odds : If in a trial there are n exhaustive events and favourable events of A are m then unfavourable events of A would be (n – m). The odd in favor of A are m : n – m and odd against of A are (n – m) : m.

$$\text{The odd in favor of event } A = \frac{m}{n-m} = \frac{\frac{m}{n}}{\frac{n-m}{n}} = \frac{P(A)}{P(\bar{A})}$$

$$\text{The odd against of event } A = \frac{n-m}{m} = \frac{\frac{n-m}{n}}{m/n} = \frac{P(\bar{A})}{P(A)}$$

Theorem : In random trial, for any event A, prove that

$$P(\bar{A}) = 1 - P(A).$$

Proof : If in a trial there are n exhaustive events and favourable events of A are m then unfavourable events of a would be $n - m$

The probability that event A will not happen.

$$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

Notation :

(i) $P(A)$ = The probability of happening an event A.

(ii) $P(\bar{A})$ = The probability of not happening an event A

(iii) $P(A+B)$ or $P(A \cup B)$ = The probability of happening at least one from A or B (if A and B are not mutually exclusive then probability of happening A or B or both)

(iv) $P(AB)$ or $P(A \cap B)$ The probability of happening both events A and B at a time.

(v) $P\left(\frac{A}{B}\right)$ = The conditional probability of event A when the event B has already happened.

(vi) $P(\bar{A}\bar{B})$ or $P(\bar{A} \cap \bar{B})$ = The probability of non-happening of the event A and B.

(vii) $P(A\bar{B} + \bar{A}B)$ = The probability of happening exactly one from A or B (But not A and B both).

ILLUSTRATIVE EXAMPLES

Example 1 : Find the probability of throwing an even number with a die.

Solution : In a throw of a die, 6 types of numbers can occur. Hence the number of exhaustive events = 6, even number 2, 4, 6 will occur for the required event, which is in number 3. So number f favorable events = 3.

$$\therefore \quad \text{Required probability} = \frac{3}{6} = \frac{1}{2}$$

Example 2 : In a single throw of two dice, determine probability of getting a total of 7.

Solution : On throwing two dice $6 \times 6 = 36$ output can be obtained. So exhaustive cases for required event = 36.

The following pairs are possible for a total of 7 : (1, 6), (2, 5), (3, 4), (4, 3) (5, 2), (6, 1) which number is 6.

So favourable number of events = 6

$$\therefore \quad \text{Required probability} = \frac{6}{36} = \frac{1}{6}$$

Example 3 : Find the probability that a leap year, selected at random, will contain 53 Mondays.

Solution : We known that in a leap year there are 366 days. So 52 complete weeks and 2 days are remaining. The seven possibilities of these 2 days are as follows :

(1) Sunday and Monday

(2) Monday and Tuesday
(3) Tuesday and Wednesday
(4) Wednesday and Thursday
(5) Thursday and Friday
(6) Friday and Saturday
(7) Saturday and Sunday.

So exhaustive cases for required event = 7

Out of seven possible cases in two cases Monday occurs. So favorable cases for required event = 2

$$\therefore \quad \text{Required probability} = \frac{2}{7}$$

Example 4 : From 12 tickets marked 1 to 12, if one ticket is selected at random, find the probability that the number on it is a multiple of 2 or 3.

Solution : The multiples of 2 or 3, in number 1 to 12 are 2, 3, 4, 6, 8, 9, 10, 12.

So out of 12 equally likely cases 8 are favourable.

$$\therefore \quad \text{Required probability} = \frac{8}{12} = \frac{2}{3}$$

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Example 5 : From a pack 52 cards, two cards are drawn at random. Prove that the probability that both cards are jack is $\frac{1}{221}$.

Solution : The exhaustive cases in which 2 cards can be drawn from a pack of 52 cards = ${}^{52}C_2$. The favourable cases in which 2 Jacks can be selected out of 4 cards = ${}^4C_2$

$$\therefore \quad \text{Required probability} = \frac{{}^4C_2}{{}^{52}C_2} = \frac{\frac{4\times 3}{2\times 1}}{\frac{52\times 51}{2\times 1}}$$

$$= \frac{4\times 3}{2\times 1} \times \frac{2\times 1}{52\times 51} = \frac{1}{221}$$

Example 6 : Three coins are tossed together then find the probability that (i) only two tails occur and (ii) at least two tails occur.

Solution : The number of exhaustive cases on tossing 3 coins = $2^3 = 8$

[*HHH, HHT, HTH, THH, HTT, THT, TTH, TTT*]

(i) Favourable cases that only 2 tails occur = 3

$$\therefore \quad \text{Required probability} = \frac{3}{8}$$

(ii) Favourable cases that at least two tails occur = 4

Important

$$\text{Required probability} = \frac{4}{8} = \frac{1}{2}$$

Example 7 : A bag contains 3 white and 5 black balls. If two balls are drawn at random, then find the odd in favor of both balls being black.

Solution : Total number of balls in bag = 3 + 5 = 8 **[R.U. 2015]**

The exhaustive case of 2 balls drawn out of 5 black balls = ${}^5C_2 = 10$

$\therefore$ Unfavourable cases = 28 − 10 = 18

So odd in favor of event = favourable cases : Unfavorable cases

= 10 : 18 = 5 : 9

Example 8 : Four persons are chosen at random from a group of 4 men, 3 women and 5 children. Find the probability that in selected persons exactly two will be children.

Solution : Total persons = 4 + 3 + 5 = 12

The exhaustive cases of 4 person chosen out of 12 person = ${}^{12}C_4$

If in each case exactly two are children, such selections can be made in ${}^5C_2$ ways with two children, the rest two can be chosen from 7 persons (4 men + 3 women) whose way of selections is ${}^7C_2$.

Hence favourable cases for required selection = ${}^5C_2 \times {}^7C_2$

$$\therefore \quad \text{Required probability} = \frac{{}^5C_2 \times {}^7C_2}{{}^{12}C_4} = \frac{5!}{2!\,3!}$$

$$= \frac{\frac{5\times 4\times 7\times 6}{2\times 1\times 2\times 1}}{\frac{12\times 11\times 10\times 9}{4\times 3\times 2\times 1}} = \frac{5\times 7\times 6}{11\times 5\times 9} = \frac{14}{33}$$

EXERCISES 12.1

1. In a throw of a die, determine the probability of getting a number more then four.
2. A coin is tossed twice, find the probability of getting tails both times.
3. One number is selected at random from natural numbers from 1 to 17. Find the

probability that the number is prime.

4. Find the probability of throwing head or tail alternatively in 3 successive tossing of a coin.
5. In a single throw of two dice, find the probability that a doublet or a total of 9 will appear. [we add all probability]
6. Find the probability that a non-leap year should have only 52 Sundays.
7. A card is drawn from a pack of 52 cards. Find the odds in favor of that card being an ace.
8. In a class of 12 students, 5 are boys and rest are girls. Find the odds against that student being a girl.
9. A party of n persons sit on a round table, find the odds against two specified individuals sitting next to each other.
10. Three letters to each of which corresponds an envelcope are placed in the envelopes at random. What is the probability that all letters, arc not placed in the right envelope.
11. Find the probability that an integer chosen at random from the first 200 positive integers is divisible by 6 or 8.
12. In a single throw of three dice, determine the probability of getting a total of more than 15.
13. The letters of word 'ANGLE' are placed at random in a row. Find the probability that vowels come together.
14. A card is drawn from a deck of 52 cards. Find the probability of getting an ace, or a king or a queen.
15. A bag contains 6 white, 7 red and 5 black balls. Out of them three balls are drawn at random one by one without replacement. Find the probability of getting all white balls.

ANSWERS 12.1

1. $\frac{1}{3}$	2. $\frac{1}{4}$	3. $\frac{7}{17}$	4. $\frac{1}{4}$
5. $\frac{5}{18}$	6. $\frac{6}{7}$	7. 1 : 12	8. 5 : 7
9. $\frac{n-3}{2}$	10. $\frac{5}{6}$	11. $\frac{1}{4}$	12. $\frac{5}{108}$
13. $\frac{2}{5}$	14. $\frac{3}{13}$	15. $\frac{5}{204}$	

Addition theorem of probability of theorem of total probability [R.U. 2016]

WHEN EVENTS ARE MUTUALLY EXCLUSIVE

Theorem 1 : The probability that one of two mutually exclusive events shall happen is the sum of the probabilities of the separate events. If A and B are two mutually exclusive events, then

$$P(A + B) = P(A) + P(B)$$

or

$$P(A \cup B) = P(A) + P(B)$$

Proof : Let exhaustive cases of events be n and favourable cases of events A and B are m_1 and m_2 respectively

$$P(A) = \frac{m_1}{n}$$

$$P(B) = \frac{m_2}{n}$$

Since the events A and B are mutually exclusive, therefore the favourable cases will all be different and favourable cases to any one of the event A and B be $m_1 + m_2$.

$$P(A + B) = \frac{m_1 + m_2}{n} = \frac{m_1}{n} + \frac{m_2}{n} = P(A) + P(B)$$

$$P(A + B) = P(A) + P(B)$$

Generatisation : The probability that one of n mutually exclusive events shall happen is the sum of the probabilities of the separate events, i.e.....

$$P(A_1 + A_2 + A_3 + ... + A_n) = P(A_1) + P(A_2) + P(A_3) + ...+P(A_n)$$

when events are not mutually exclusive.

Theorem 2 : If A and B are two mutually inclusive events then the probability of happening any one of them is as follows :

$$P(A + B) = P(A) + P(B) - P(AB)$$

or

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof : Let exhaustive cases of events be n and favourable cases of events A and B be m_1 and m_2 respectively.

$$\therefore \quad P(A) = \frac{m_1}{n}, P(B) = \frac{m_2}{n}$$

Since A and B are any event so it is possible that they are not mutually exclusive. So there may be some common events. Let m_3 be the common favourable events in A and B.

$$P(AB) = \frac{m_3}{n}$$

Favourable cases of events $(A + B)$ is $m_1 + m_2 - m_3$

$$\therefore \quad P(A + B) = \frac{m_1 + m_2 - m_3}{n} = \frac{m_1}{n} + \frac{m_2}{n} - \frac{m_3}{n}$$

or $$P(A + B) = P(A) + P(B) - P(AB)$$

or $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Corollary : If the events are mutually exclusive then

then $$A \cap B = \phi \text{ and } P(A \cap B) = 0$$

$$\therefore \quad P(A \cup B) = P(A) + P(B)$$

or $$P(A + B) = P(A) + P(B)$$

Multiplication theorem of probability or theorem of compound probability [R.U. 2016]

Probability of simultaneous occurrence of two events A and B is equal to the probability of A multiplied by the conditional probability of B on the assumption that A has happened or probability of B multiplied by the conditional probability of A on the assumption that B has happened.

$$P(AB) = P(A).\ P(B|A)$$

or $$P(A \cap B) = P(A).\ P(B|A)$$

$$P(AB) = P(B).P(A|B)$$

or $$P(A \cap B) = P(B).P(A|B)$$

Proof : Let n be that total number of mutually exclusive and equally likely cases out of which m cases are governable to the event A. Let m_1 be favourable cases to both the events A and B. Then m_2 is included in m cases favourable to A.

$$P(AB) = \frac{m_1}{n} = \frac{m_1}{m} \times \frac{m}{n}$$

But $$P(A) = \frac{m}{n}$$

Similarly we can prove that

$$P(AB) = P(B).\ P(A|B)$$

or $$P(AB) = P(A|B).P(B)$$

$$P(AB) = P(B).P(A|B)$$

Corollary : A and B are independent events, then

$$P(B|A) = P(B)$$

$$P(AB) = P(A).P(B)$$

Generalisation : If $A_1, A_2 \ldots\ldots A_n$ are independent events then $P(A_1 . A_2 . A_3 \ldots . A_n) = P(A_1).\ P(A_2),\ P(A_3) \ldots\ldots p(A_n)$

Probability of at least one event

If probability of n independent events are $P_1, P_2, \ldots, P_n$ respectively, then find the probability of at least one of the events will happen.

Let $A_1, A_2 \ldots A_n$ be independent events whose probabilities are $P_1, P_2 \ldots\ldots P_n$ respectively then $P(A_1) = P_1, P(A_2) = P_2 \ldots\ldots P(A_n) = P_n$ and $P(\bar{A}_1) = 1 - P_1$,

$P(\bar{A}_2) = 1 - P_2 \ldots\ldots\ P(\bar{A}_n) = 1 - P_n$.

Since $A_1, A_2 \ldots\ldots A_n$ are independent events so $\bar{A}_1, \bar{A}_2 \ldots\ldots \bar{A}_n$ are also independent events.

Therefore probability of none of the events will happen (by multiplication theorem)

$= P(\bar{A}_1 . \bar{A}_2 \ldots . \bar{A}_n)$

$$= P(\bar{A}_1)P(\bar{A}_2) \ldots\ldots P(\bar{A}_n)$$

$$= (1 - P_1)(1 - P_2) \ldots (1 - P_n)$$

Hence the probability that at test one event will happen is

= 1 – probability of none of the event will happen

$$= 1 - P(\bar{A}_1)P(\bar{A}_2) \ldots\ldots P(\bar{A}_n)$$

$$= 1 - [(1 - P_1)(1 - P_2) \ldots\ldots (1 - P_n)]$$

Ecs9

Illustrative examples

☐ **Example 1 : In a single throw of two dice, determine the probability of getting a total of 7 or 11.**

Solution : The number of exhaustive cases on throwing 2 dice

$$= 6 \times 6 = 36$$

Favourable cases for a total of 7

$= (6, 1), (5, 2), (4, 3), (3, 4), (2, 5)\ (1, 6) = 6$

$\therefore \quad P(7) = \frac{6}{36}$

Favorable cases for a total of 11

$(6, 5)\ (5, 6) = 2$

$\therefore \quad P(11) = \frac{2}{36}$

$\therefore$ Total probability because events are mutually exclusive,

$$P(7+11) = P(7) + P(11) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$$

easy

□ Example 2 : A bag contains 2 white, 4 black and 5 red balls. Three balls are drawn at random. Find the probability that all three balls are of same colour.

Solution : In the bag total number of balls $2 + 4 + 5 = 11$

total ways of drawing 3 balls = ${}^{11}C_3$

All three balls of same colour can be red or black :

Probability of all three balls being red = $\frac{{}^5C_3}{{}^{11}C_3} = \frac{10}{165}$

Probability or all three balls being black = $\frac{{}^4C_3}{{}^{11}C_3} = \frac{4}{165}$

$\because$ These two events are mutually exclusive.

Hence required probability = $\frac{10}{165} + \frac{4}{165} = \frac{14}{165}$

□ Example 3 : A card is drawn at random from a will shuffled deck of 52 cards. Find the probability of its being an are or a heart card.

Solution : Let the event that drawn card be an ace be denoted by A and event that card be a heart is denoted by B. Here A and B are not mutually exclusive events because if card drawn is an ace of heart then both evénts happen at a time, so by addition theorem of probability.

$$P(A+B) = P(A) + P(B) - P(AB)$$

The exhaustive cases for event $A = {}^{52}C_1 = 52$ and number of aces in a pack is 4, for which favourable cases = ${}^4C_1 = 4$

$\therefore \quad P(A) = \frac{4}{52} = \frac{1}{13}$

The exhaustive cases for event $B = {}^{52}C_1 = 52$

favourable cases for event $\quad B = {}^{13}C_1 = 13$ (In a pack, heart cards are 13)

$\therefore \quad P(B) = \frac{13}{52}$

Favourable cases of happening event A and B together = 1

$\therefore \quad P(AB) = \frac{1}{52}$ (When ace is of heart)

$$P(A+B) = P(A) + P(B) - P(A.B)$$

$$P(A+B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

□ Example 4 : A, B, C are participating in different competitions. The probability of getting success of A is $\frac{2}{5}$, B is $\frac{1}{8}$ and C is $\frac{5}{8}$. Find the probability that (i) all three get success (ii) at least one gets success.

Solution : Here $P(A) = \frac{2}{5}$ so $P(\bar{A}) = 1 - \frac{2}{5} = \frac{3}{5}$

(A∪B)

$$P(B) = \frac{1}{8} \text{ So } P(\bar{B}) = 1 - \frac{1}{8} = \frac{7}{8}$$

$$P(C) = \frac{5}{8} \text{ so } P(\bar{C}) = 1 - \frac{5}{8} = \frac{3}{8}$$

(i) Events of getting success are independent so probability of all three getting success by rule of compound probability

$$= \frac{2}{5} \cdot \frac{1}{8} \cdot \frac{5}{8} = \frac{1}{32}$$

No success + At least one succ

(ii) Probability that at least one gets success

$$= 1 - P\,P(\bar{A}\bar{B}\bar{C})$$

= 1

$$= 1 - P(\bar{A}).P(\bar{B}).P(\bar{C})$$

P(A) + P(Ā)

$$= 1 - \frac{3}{5} \cdot \frac{7}{8} \cdot \frac{3}{8}$$

$$= 1 - \frac{63}{320}$$

$$= \frac{257}{320}$$

most

☐ **Example 5 : Mohan speaks truth in 60% of the cases, Sohan speaks truth in 80% of the cases. What is the probability that they are likely to contradict each other in stating the same fact ?**

Solution : Let A and B denote the event of truth speaking of Mohan and Sohan respectively then

$$P(A) = \frac{60}{100} = \frac{3}{5}$$

$$P(\bar{A}) = 1 - \frac{3}{5} = \frac{2}{5}$$

$$P(B) = \frac{80}{100} = \frac{4}{5}$$

$$\therefore \quad P(\bar{B}) = 1 - \frac{4}{5} = \frac{1}{5}$$

(i) Mohan speaks truth and Sohan tells a lie = $A\bar{B}$

(ii) Mohan tells a lie and Sohan speaks truth = $\bar{A}B$

Since $A\bar{B}$ and $\bar{B}A$ are independent events.

$$\therefore \quad P(\bar{A}B) = P(\bar{A}).P(B) = \frac{2}{5} \times \frac{4}{5} = \frac{8}{25}$$

$$P(A\bar{B}) = P(A).P(\bar{B}) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$$

Again $A\bar{B}$ and $\bar{A}B$ are mutually exclusive events

$$\therefore \quad P(\bar{A}B + A\bar{B}) = P(\bar{A}).P(B) + P(A).P(\bar{B})$$

$$= \frac{8}{25} + \frac{3}{25} = \frac{11}{25}$$

☐ **Example 6 : In a class it is known by seeing mark sheets of students that 40% students have passed in Mathematics, 25% students have passed in Physics and 15% students have passed in Mathematics and Physics both. If a student has passed in Mathematics then find the probability that this student has passed in Physics.**

Solution : Let selected student will pass in Mathematics be [illegible] Physics be event B. Then it is given

$$P(A) = \frac{40}{100} = \frac{2}{5} \text{ and } P(B) [illegible]$$

and

$$P(AB) = \frac{15}{100} = \frac{3}{20}$$

Now we have to find $P(B/A)$ because if selecte[illegible] mathematics then probability of these students, also p[illegible] multiplication theorem of probability.

$$P(AB) = P(A).\, P[illegible]$$

or

$$\frac{3}{20} = \frac{2}{5} \cdot P(B/A)$$

$$\therefore \quad \text{Required probability } P(B/A) = \frac{3}{20} \times \frac{5}{2} = \frac{3}{8}$$

☐ **Example 7 : A and B throw alternatively with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, prove that the probability of A's winning is $\frac{30}{61}$.**

Solution : Let a total of 6 occurring on two dice be event E_1.

The exhaustive case for event $E_1 = 6^2 = 36$

and favourable cases are (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1) i.e., total favourable cases are 6.

$$\therefore \quad P(E_1) = \frac{5}{36} \text{ so } P(\bar{E}_1) = 1 - \frac{5}{36} = \frac{31}{56}$$

Again let a total of 7 occurring on two dice be event E_2.

The exhaustive cases for event $E_2 = 36$

and favourable cases = (6, 1), (5,2), (4, 3), (3, 4), (2, 5) and (1, 6).

i.e. total favourable cases are 6.

$$\therefore \quad P(E_2) = \frac{6}{36} = \frac{1}{6} \text{ So } P(\bar{E}_2) = 1 - \frac{1}{6} = \frac{5}{6}$$

If A begins the game then chance of his winning:

(i) Probability of winning in first attempt $P(E_1) = \frac{5}{36}$

...es not occur in first attempt, 7 does not occur in B's first attempt and ...ccurring in A's second attempt and probability of $\bar{E}_1.\bar{E}_2.E_1$

$$P(\bar{E}_1,\bar{E}_2,E_1) = P(\bar{E}_1).P(\bar{E}_2).P(E_1)$$

$$= \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}$$

...milarly probability can be calculated for further attempts.

□ Ex... truth in ... probability of A's winning

$$= \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + ...$$

$$= \frac{\frac{5}{36}}{1 - \frac{31}{36} \times \frac{5}{6}} = \frac{30}{61}, \text{ (by sum formula of infinite G.P.)}$$

RQ

□ **Example 8 : A book is reviewed by three critics. Odds in favour of the book are 5 : 2, 4 : 3 and 3 : 4 respectively for three critics. Find the probability that the majority are in favour of the book.** **[R.U. 2015]**

Solution : Let E_1, E_2 and E_3 denote the events that the book will be reviewed favourably by the first, the second and the third critic respectively. We are given that

$$P(E_1) = \frac{5}{7}, P(E_2) = \frac{4}{7}\ P(E_3) = \frac{3}{7}$$

$$\therefore \quad P(\bar{E}_1) = \frac{2}{7}, P(\bar{E}_2) = \frac{3}{7}, P(\bar{E}_3) = \frac{4}{7}$$

The cases that book will be favourably reviewed by the majority of the reviewers is as follows :

1. $E_1E_2E_3$ 2. $\bar{E}_1E_2E_3$ 3. $E_1\bar{E}_2E_3$ 4. $E_1E_2\bar{E}_3$

whose respective probability will be

$$P(E_1E_2E_3) = P(E_1)\ P(E_2)\ P(E_3) = \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{60}{343}$$

$$P(\bar{E}_1E_2E_3) = P(\bar{E}_1)\ P(E_2)\ P(E_3) = \frac{2}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{24}{343}$$

$$P(E_1\bar{E}_2E_3) = P(E_1)\ P(\bar{E}_2)\ P(E_3) = \frac{5}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{45}{343}$$

$$P(E_1E_2\bar{E}_3) = P(E_1)\ P(E_2)\ P(\bar{E}_3) = \frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{80}{343}$$

Above events are mutually exclusive. Hence required probability

$$P(E_1E_2E_3) + P(\bar{E}_1E_2E_3) + P(E_1\bar{E}_2E_3) + P(E_1E_2\bar{E}_3)$$

$$= \frac{60}{343} + \frac{24}{3434} + \frac{45}{343} + \frac{80}{343} = \frac{209}{343}$$

question RU

□ **Example 9 : A bag contains 6 red and 4 white balls. Two draws of 2 balls are made from bag. Find the probability that the first draw gives 2 red balls and the second draw 2 white balls.**

(i) If the balls are replaced in the bag after the first draw.

(ii) If the balls are not replaced.

Solution : (i) When balls are replaced in the bag :

Total balls in the bag = 6 + 4 = 10

$\therefore$ Ways of drawing 2 balls from the bag = $^{10}C_2$

Total ways of drawing 2 balls from 6 red balls in first draw = $^6C_2$

$\therefore$ Probability of getting 2 red balls in first draw = $\frac{^6C_2}{^{10}C_2}$

Ways of drawing 2 balls from 4 white balls = $^4C_2$

$\therefore$ Probability of getting 2 white balls in second draw = $\frac{^4C_2}{^{10}C_2}$

Above events are independent, hence

Required probability = $\frac{^6C_2}{^{10}C_2} \times \frac{^4C_2}{^{10}C_2} = \frac{1}{3} \times \frac{2}{15} = \frac{2}{45}$

(ii) When balls are not replaced in bag in second draw balls left in the bag is = 10 − 2 = 8

Probability of getting 2 white balls in second draw = $\frac{^4C_2}{^8C_2}$

Hence required probability = $\frac{^6C_2}{^{10}C_2} \times \frac{^4C_2}{^8C_2} = \frac{1}{3} \times \frac{3}{14} = \frac{1}{14}$

RJ

□ **Example 10 : The odds against an event are 5 : 9 and the odds in favour of another event are 4 : 3. Find the probability that exactly are of the two events will occur.** **[R.U. 2016]**

Solution : Probability of first event P(A) = $\frac{9}{14}$

Probability of second event $P(B) = \frac{4}{7}$

Probability that exactly one of the two events will occur

$$= P(A)P(\bar{B}) + P(\bar{A})P(B)$$

$$= \frac{9}{14}\cdot\frac{3}{7} + \frac{5}{14}\cdot\frac{4}{7}$$

$$= \frac{27}{98} + \frac{20}{98}$$

$$= \frac{47}{98}$$

Example 11 : A can solve 4 problems out of 6, B can solve 5 problems out of 6 and C can solve 2 problems out of 5. If they all try to solve a problem, then find the probability that at least 2 of them solve this problem. [R.U. 2016]

Solution : $P(A) = \frac{4}{6},\ P(B) = \frac{5}{6},\ P(C) = \frac{2}{5}$

Probability that at least 2 of them solve the problem

$$= P(AB\bar{C}) + P(AC\bar{B}) + P(\bar{A}BC) + P(ABC)$$

$$= \frac{4}{6}\cdot\frac{5}{6}\cdot\frac{3}{5} + \frac{4}{6}\cdot\frac{2}{5}\cdot\frac{1}{6} + \frac{2}{6}\cdot\frac{5}{6}\cdot\frac{2}{5} + \frac{4}{6}\cdot\frac{5}{6}\cdot\frac{2}{5}$$

$$= \frac{60}{180} + \frac{8}{180} + \frac{20}{180} + \frac{40}{180}$$

$$= \frac{128}{180} = \frac{32}{45}$$

Example 12 : A bag contains 4 red, 6 yellow, 3 pink balls. If 3 balls are drawn at random, find the probablity in following cases : [R.U. 2016]

(i) All are red

(ii) All are pink

(iii) No one is yellow

(iv) One ball of each color

(v) At least one yellow ball

(vi) The balls are drawn one by one in the order yellow, red and pink.

Solution : Total number of balls = 13

Total ways of drawing 3 balls at random = $^{13}C_3$

1. Probability that all balls are red = $\frac{^4C_3}{^{13}C_3}$

$$= \frac{4\times3\times2}{13\times12\times11} = \frac{2}{143}$$

2. Probability that all are pink $= \frac{^3C_3}{^{13}C_3}$

$$= \frac{1\times3.2.1}{13.12.11} = \frac{1}{286}$$

3. No ball is yellow = 2 Red 1 pink or 1 Red 2 pink or 3 pink or 3 Red balls.

$$\text{Probability} = \frac{^4C_2\times{}^3C_1 + {}^4C_1\times{}^3C_2 + {}^4C_3 + {}^3C_3}{^{13}C_3}$$

$$= \frac{(18+12+4+1)\times3\times2\times1}{13\times12\times11}$$

$$= \frac{35\times3\times2\times1}{13\times12\times11}$$

$$= \frac{35}{286}$$

4R 3P

2R+1P

1R+2P

3R

3P

Alternate method : 3 balls out of 7 non yellow balls

$$\text{Probability} = \frac{^7C_3}{^{13}C_3} = \frac{35}{286}$$

4. One ball of each colour = $\frac{^4C_1\times{}^6C_1\times{}^3C_1}{^{13}C_3}$

$$= \frac{4\times6\times3\times3\times2\times1}{13\times12\times11}$$

$$= \frac{36}{143}$$

5. At least one yellow ball = 1 – P(No yellow ball)

$$= 1 - \frac{35}{286} = \frac{251}{286}$$

6. The balls are drawn one by one in order yellow, red and pink

$$= \frac{6}{13}\times\frac{4}{12}\times\frac{3}{11}$$

$$= \frac{6}{143}$$

EXERCISES 12.2

1. A coin is tossed four times. Find the probability of getting at least 3 heads in these tosses.
2. A coin is tossed and a die is thrown. Then find the probability that the outcome will be a head on coin and an even number on die.
3. A man can kill a birds once in three shots. On this assumption he fires three shots. What is the probability that the bird is killed ?
4. Two faces of die are red, two are black and two are yellow. If it is thrown twice then what is the probability that both faces are of same colour ?
5. In a company of 20 persons, 5 are graduates. If 3 person are selected at random then what is the probability that at least one of them is a graduate.
6. The odds against A, solving a certain problem are 4 to 3, odds in favour of B solving the same problem are 7 to 5. What is the chance that (i) the problem will be solved (ii) The problem will not be solved (iii) the problem will be solved only by one.
7. A purse contains 4 coins of Rs 5 and 9 coins of Rs 2. Another purse contains 6 coins of Rs 5 and 7 coins of Rs. 2. A purse is chosen at random and a coin is drawn from it. What is the probability that it is a coin of Rs. 5 ?
8. X and Y loss a coin alternately till one of them gets a head and wins the game. If X starts the game find the probabilities of both winning.
9. A bag contains 3 red and 3 blue balls. Two balls are drawn from bag one by one without replacement. Find the probability that both balls are of different colour.
10. A piece of equipment will function only when all the three component A, B and C are working. The probability of A failing during one year is 0.15, that of B is 0.05 and that of C is 0.10. What is the probability that the equipment will cail before the end of the year ?
11. In a pack of cards two cards are drawn at random two times. If drawn cards of first time are not replaced in the pack, then find the probability that two aces occur in first draw and two kings occur in second draw.
12. A and B are two events in which $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A.B) = \frac{1}{12}$ find $P(B/A)$.

$P(\frac{B}{A}) = \frac{P(A \cap B)}{A}$

ANSWERS 12.2

1. $\frac{5}{16}$ 2. $\frac{1}{4}$ 3. $\frac{19}{27}$ 4. $\frac{1}{3}$

5. $\frac{137}{228}$ 6. $\frac{16}{21}, \frac{5}{21}, \frac{43}{84}$ 7. $\frac{5}{13}$ 8. $\frac{2}{3}, \frac{1}{3}$

9. $\frac{3}{5}$ 10. 0.27325 11. $\frac{6}{270725}$ 12. $\frac{1}{4}$